Disentangling Bohr and Heisenberg: complementarity is a stronger feature than disturbance

University of Gdańsk

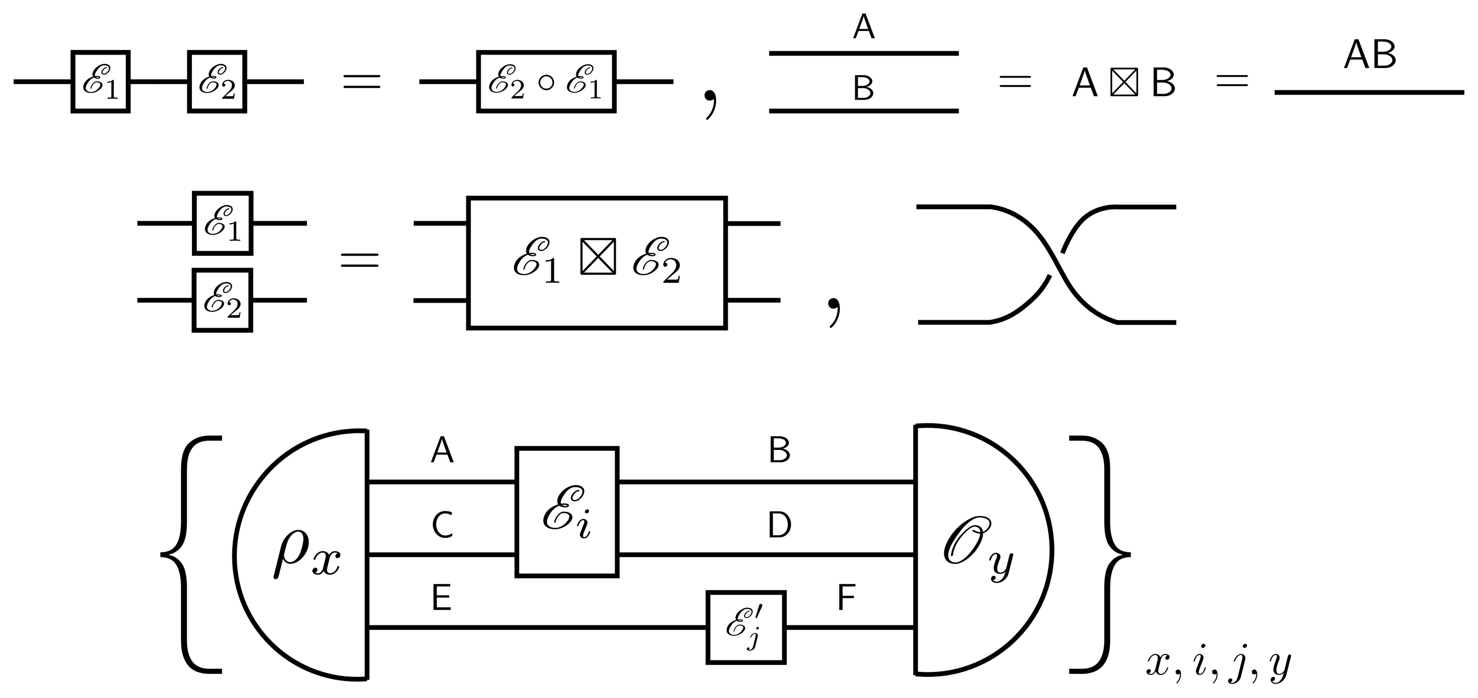
MARCO ERBA*, Paolo Perinotti†, Davide Rolino†, Alessandro Tosini†
*International Centre for Theory of Quantum Technologies (ICTQT), Gdańsk, Poland
†University of Pavia (Department of Physics) & INFN, QUit group, Pavia, Italy

International Centre for Theory of Quantum Technologies

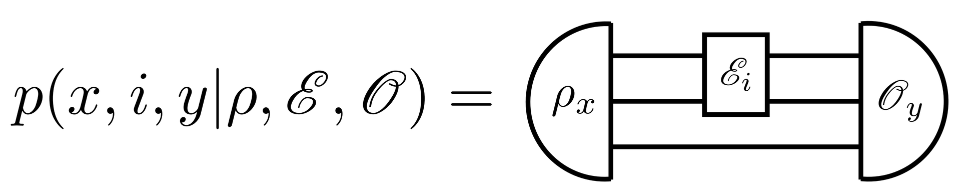
The possibility of disturbing a system à la Heisenberg, as put forward by his *Gedankenexperiment*, does not imply complementarity. A complete theory, called Minimal Classical Theory (MCT), is constructed as a counterexample. MCT also allows to prove that incompatibility is a stronger feature than disturbance.

Operational Probabilistic Theories

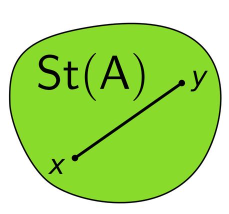
WHAT DO WE MEAN BY (OPERATIONAL) *THEORY*?



PHYSICAL SYSTEMS, EVENTS... viz. *EXPERIMENTS*

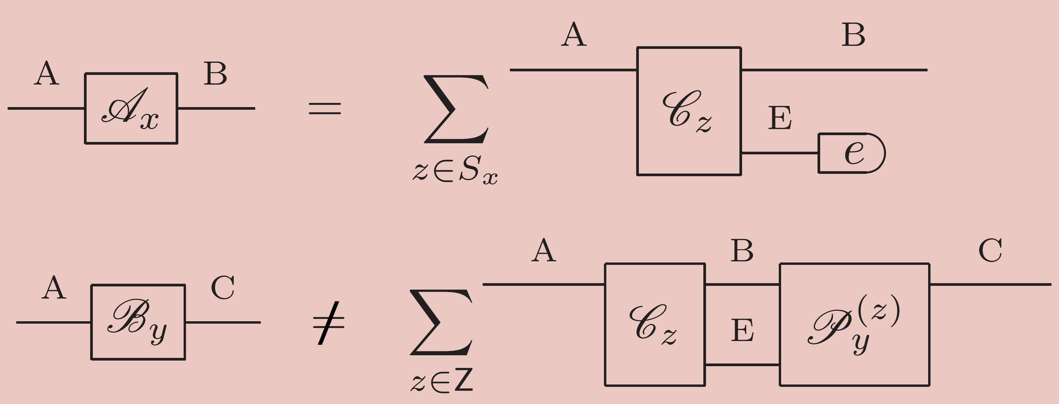


States–events, in general–for any system A are [*contained in*] convex sets in real linear spaces; this enables a general account of randomisations, mixtures, LOCC protocols...



Disturbance action: Irreversibility

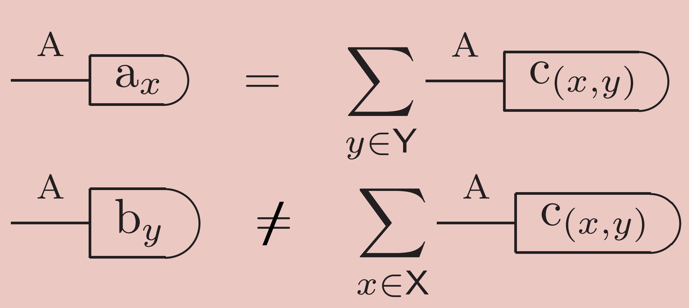
The existence of a test excluding the performance of another one:



This is analogous to thermodynamical irreversibility.

Incompatibility of observables

The impossibility of jointly measuring a pair of observables:

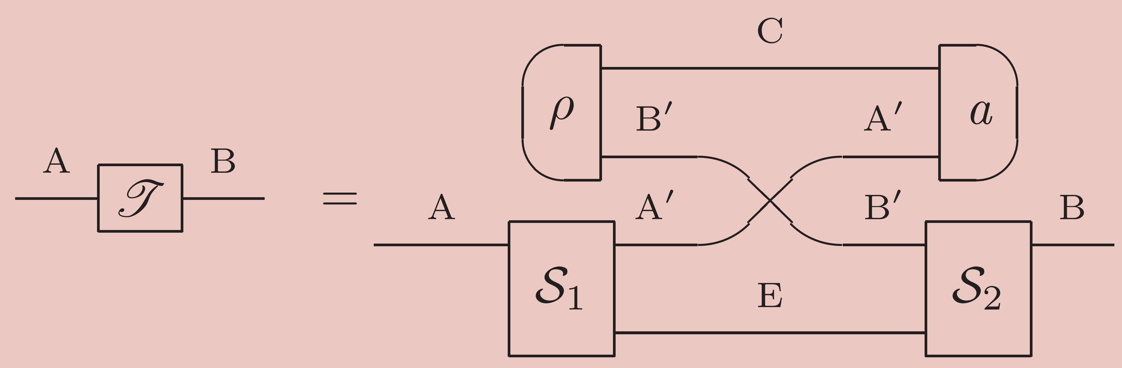


Complementarity

The existence of statements, concerning *properties* of a physical system, for which it is impossible to simultaneously assert their truth value.
This feature is traditionally associated with incompatibility of observables.

Minimal Classical Theory (MCT)

Operationally minimal version of classical information theory:



...and its Cauchy completion w.r.t. the operational norm:

$$\|\rho\|_{op} := \sup_{\{a_0, a_1\}} (a_0 - a_1 |\, \rho)$$

$$\|\mathscr{T}\|_{op} := \sup_{C} \sup_{\rho \in \mathrm{St}_1(\mathrm{AC})} \|(\mathscr{T} \boxtimes \mathscr{I}_{\mathrm{C}})\,\rho\|_{op}$$

MCT exhibits irreversibility without complementarity!

Irreversibility does *not* imply Incompatibility of observables!

On the other hand,

Incompatibility of observables => Irreversibility in every physical theory.

Why is MCT nontrivial, and why would one care?

(1) MCT is a thorough theory, equipped with a nontrivial set of transformations, and it is closed under both sequential and parallel composition.

(2) Despite being classical, it features phenomena traditionally thought as quantum, such as *no-information-without disturbance* and *incompatibility of instruments*!

(3) It is *the only theory* that can possibly exhibit all such features *without entanglement* [1,2].

(4) Closed under *limits of experimental procedures*: it provably does not converge to Classical Theory, being inherently different from the latter.

[1] Martin Plávala "All measurements in a probabilistic theory are compatible if and only if the state space is a simplex" Phys. Rev. A 94, 042108 (2016)
[2] G. M. D'Ariano, M.E., and P. Perinotti "Classical theories with entanglement" Phys. Rev. A 101, 042118 (2020)

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