Operational causality and realizable processes in the Page-Wootters framework

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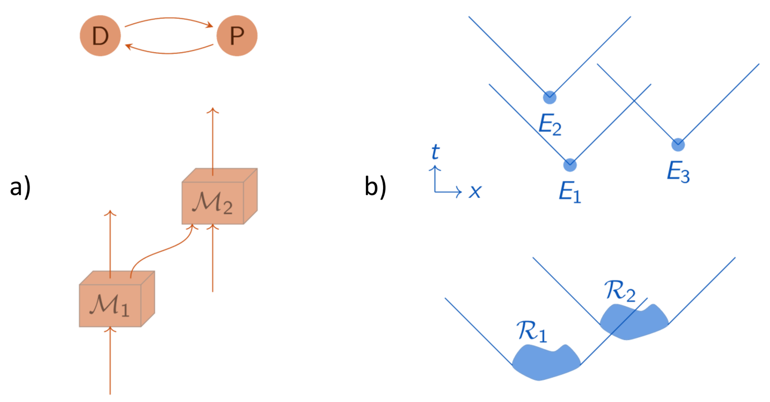
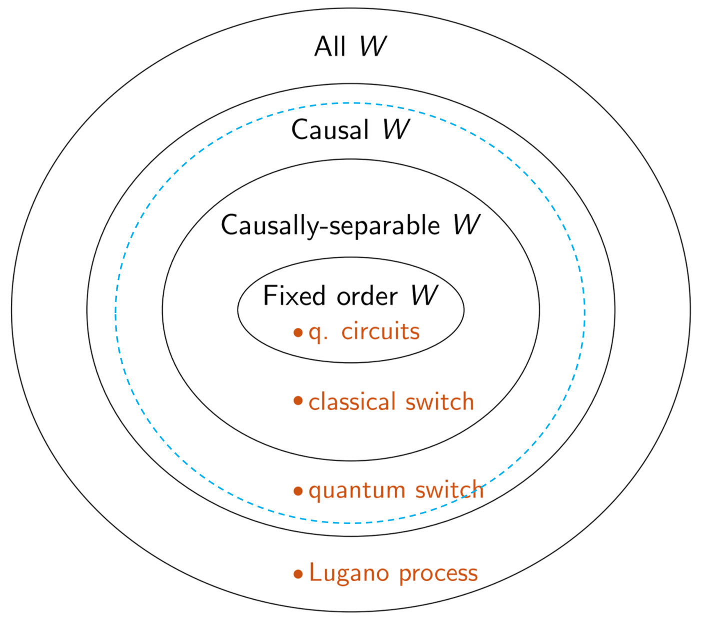
Motivation

Causality is a fundamental concept in science, but its definition varies across different disciplines. In quantum information theory, causality is associated with the flow of information a), while in relativity, it is linked to spacetime geometry b). A priori, these notions are different, but in a recent work they were made compatible [1] and used to identify possible **processes matrices** in an acyclic spacetime [2].





Information-theoretic classes of process matrices [3-6]
Process matrices compatible with a fixed spacetime [7]

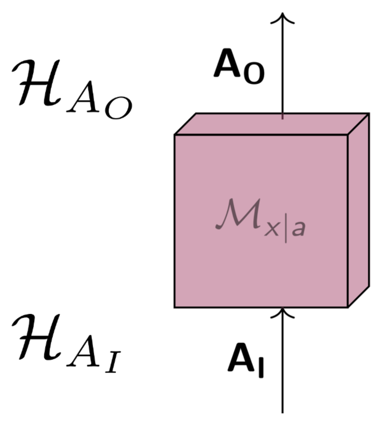
But a fixed spacetime might not be everything. Inspired by quantum gravity, we can think about situations in which a particle in superposition is causing a superposition of spacetimes. A famous example for this is the gravitational **quantum switch** [8-9], which suggests that causal structure itself can be in a superposition. One could therefore ask which kind of processes are possible if one considers superpositions of spacetime geometries or describes spatiotemporal relations relative to quantum reference frames.

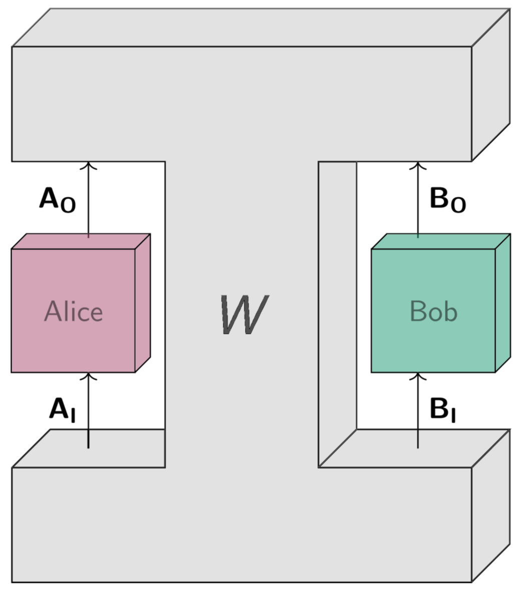
What kind of processes are possible in the Page-Wootters framework?

Process matrix framework

In every day life, all our physical experiments assume an underlying global causal structure. But what kind of processes are possible if we loosen this condition and only demand quantum mechanics to be valid locally? The **process matrix framework** [3] addresses these kind of situations in which no pre-defined global causal structure is assumed. In the framework, multiple parties sitting in **local quantum laboratories** can receive and send a quantum system to an outside environment. Additionally, parties have the opportunity to perform a unitary or measurement on their system, which can be described with quantum instruments. The **outside environment** is characterized by a **process matrix W**.

Local laboratories are modeled as input and output Hilbert space with. Each party has access to a quantum instrument consisting of a set of CP maps $\mathcal{J}_a = \{M_{x|a}\}_x$





$$W \in \mathcal{L}(\mathcal{H}_{A_I} \otimes \mathcal{H}_{A_O} \otimes \mathcal{H}_{B_I} \otimes \mathcal{H}_{B_O})$$

$$W \geq 0$$

$$tr\left[\left(M^{A_I A_O} \otimes M^{B_I B_O}\right) W\right] = 1$$

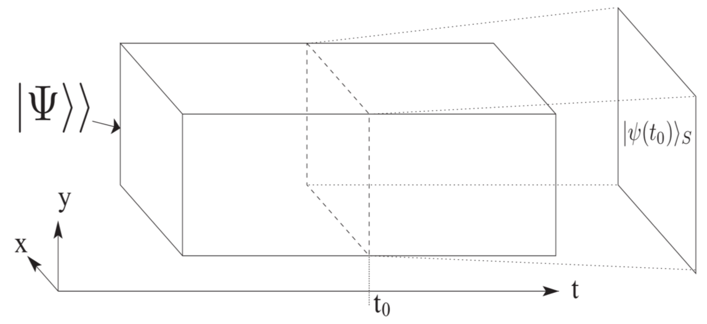
The outside environment is modeled by a **process matrix W** that fulfills the characteristics on the left. Correlations are obtained via

$$P(x, y|a, b) = tr\left[\left(M^{A_I A_O}_{x|a} \otimes M^{B_I B_O}_{y|b}\right) W\right]$$

Page-Wootters framework

In quantum mechanics time appears as a classical parameter in the Schrodinger equation. This is acceptable for all known applications, but for quantum gravity, we want to find a fully quantum description of time. However, in general it is not possible to find a time operator. A canonical quantization of general relativity leads to a constraint known as Wheeler-DeWitt equation, which implies that the wave function of the universe does not evolve in time [10]. A framework satisfying such a constraint is the Page-Wootters (PW) framework [11,12].

This framework describes quantum dynamics relative to a clock which is itself a quantum system. The PW framework involves constraint operators acting both on the physical system and the clock. Physical states (also called history states) are annihilated by the constraint operator, and describe a "block universe" which is frozen in time. The dynamical evolution of the system is recovered from the entanglement properties of the clock and the system.



$$\hat{C} = \hat{\Omega}_T \otimes \mathcal{I}_S + \mathcal{I}_T \otimes \hat{H}_S \qquad \hat{C}|\psi\rangle\rangle = 0$$

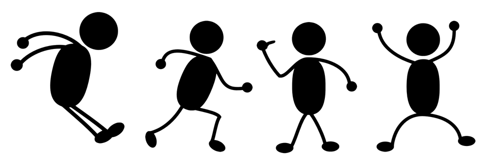
$$_T\langle t|\hat{C}|\psi\rangle\rangle = 0 \Leftrightarrow i\frac{\partial}{\partial t}|\psi(t)\rangle_S = \hat{H}_S|\psi(t)\rangle_S$$

$$\Rightarrow \quad |\psi\rangle\rangle = \int dt|t\rangle_T \otimes |\psi(t)\rangle_S$$

As Circuits: $\hat{C}|\psi\rangle\rangle = \sum_t \hat{H}_t|\psi\rangle\rangle = 0 \quad \Rightarrow \quad \sum_t |t\rangle_T \otimes |\psi(t)\rangle$

Connecting PW circuits to Process matrices

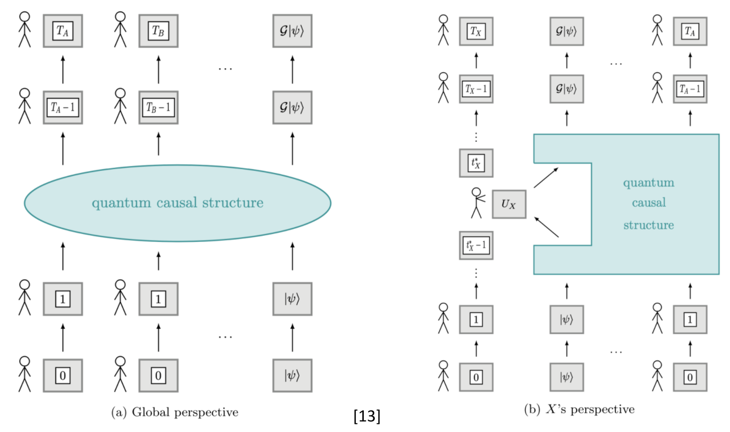
Recently, these PW circuits have been extended to process matrices [13]. This is done by extending the clock Hilbert space to multiple observers.



$$|\psi\rangle\rangle = \sum_{t_{A_1},\ldots,t_{A_N}} |t_{A_1},\ldots,t_{A_N}\rangle_T \otimes |\psi(t_{A_1},\ldots,t_{A_N})\rangle_S$$

$$|\psi(t_{A_1},\ldots,t_{A_N})\rangle_S = M(t_{A_1},\ldots,t_{A_N})|\psi(0_{A_1},\ldots,0_{A_N})\rangle_S$$

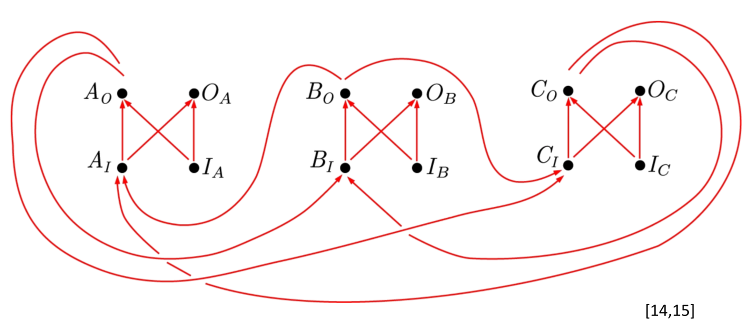
- Clocks are synchronized in past and future.
- Agents observe a unitary evolution from their perspective
- Agents only act once with their local operations



[13]

Lugano-Process

- Violates causal inequalities
- Unitary process
- Fully classical


[14,15]

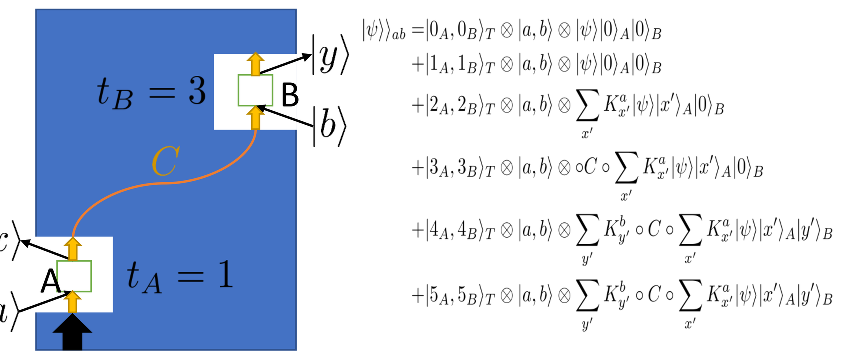
Aim

- Is the Lugano process realizable as a PW circuit?
- Investigating realizable processes
- Signaling notion in PW states
- Analyzing operational structure

Results

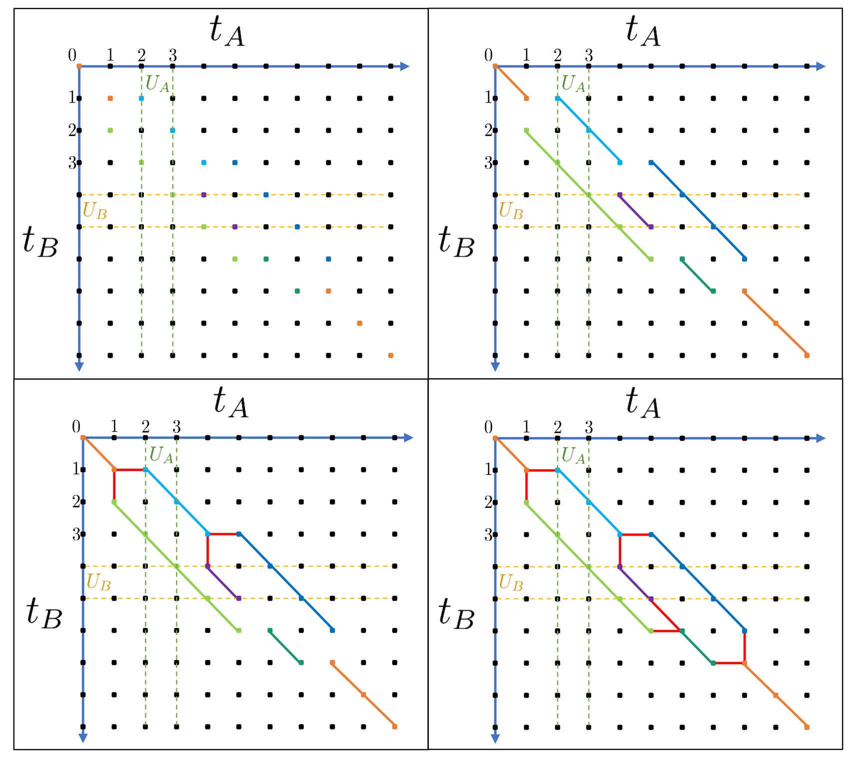
Definition of signaling in PW states

We define signaling by considering whether different choices of local operations for an agent A (encoded in the initial value of a setting variable) can lead to different measurement outcomes for another agent B. This involves including ancillas for every agent that stores their measurement result.

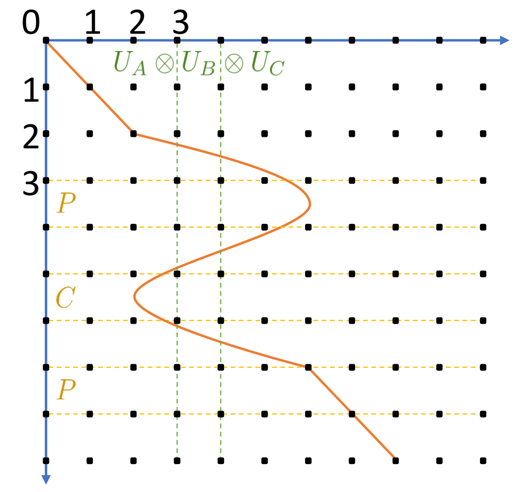


Visualizing operational order via time-reference diagrams

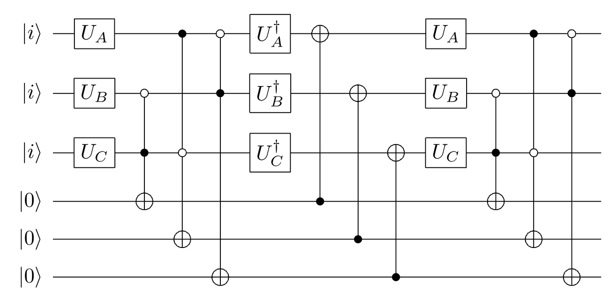
We propose a method of visualizing the history states of an N-clock scenario in terms of an N-dimensional grid. This method simplifies the analysis of the branching structure of PW states, and makes signaling relations and previously considered properties such as affine-linearity [13] visually apparent. Further analysis then suggests that under forward-moving clocks, backwards in time signaling would not be possible.



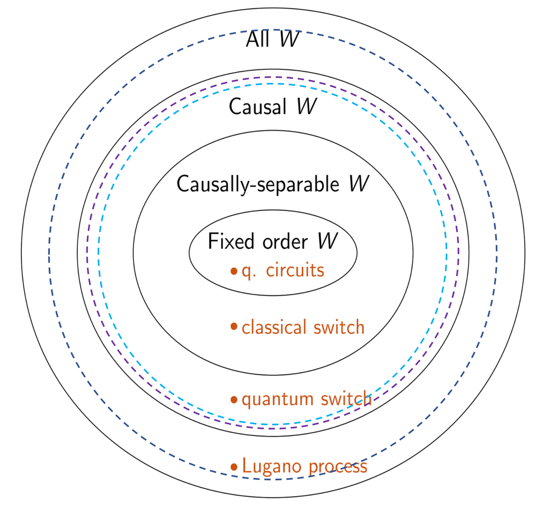
Formulating the Lugano process as PW state with backward clicking clock



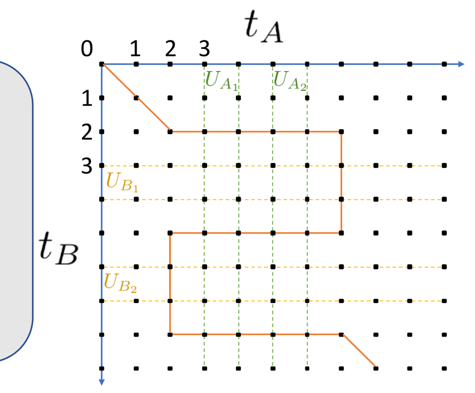
A quantum circuit with multiple local operations corresponds to a PW state with backward clicking clock and a single local operation.

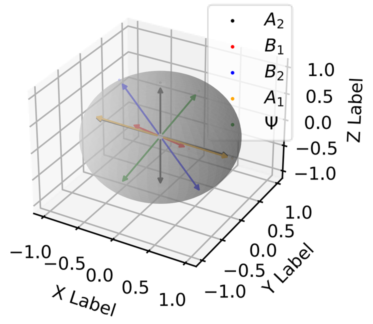


Characterizing the class of realizable processes in the strictly time increasing case



Exploring the consequence of backwards ticking clocks





By modeling measurement as unitaries, it is possible to signal back in time in this scenario

Outlook

- Mapping between PW states and result in fixed spacetime
- Extending the framework to continuous clocks
- Class of realizable processes for backwards ticking clocks
- Generalize to perspectival light cones

References

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