On circuit extraction from MBQC patterns with Pauli flow

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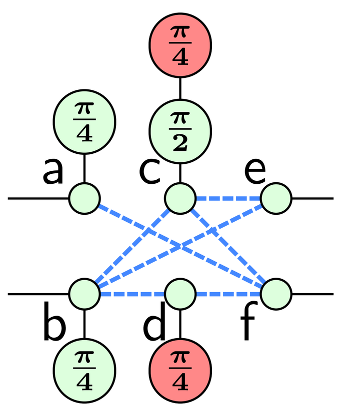
Motivation

We focus on extracting an ancilla-free quantum circuit from an MBQC pattern. So far, this is only possible if the pattern has some kind of flow, i.e. if the pattern is deterministically implementable on a quantum computer. We extend a known extraction procedure for patterns with extended gFlow [1] to patterns with the less restrictive Pauli flow. Like in the gFlow case, we do not need to calculate the actual underlying Pauli flow; it is sufficient to know that the pattern has Pauli flow. The algorithm can also be used to identify some Pauli X and Y measurements which can be relabeled as planar XY measurements without losing Pauli flow.

MBQC in ZX-calculus

Any MBQC pattern can be represented as a graph-like ZX-Diagram, where the underlying graph constructed by empty green spiders and Hadamard wires corresponds to a graph-state and the effect spiders denote measurement angles in some plane of the Bloch sphere:



$\lambda(c) = XZ$
$\lambda(d) = YZ$
XY, otherwise

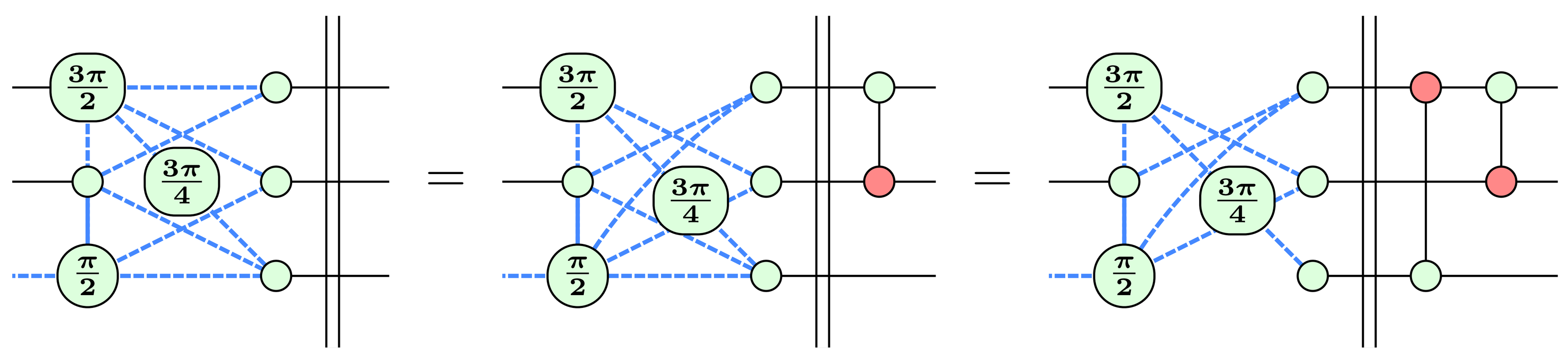
$g(a) = \{f\}$
$g(b) = \{c\}$
$g(c) = \{c, e\}$
$g(d) = \{c, d\}$

$a \prec b, d \prec c \prec e, f$

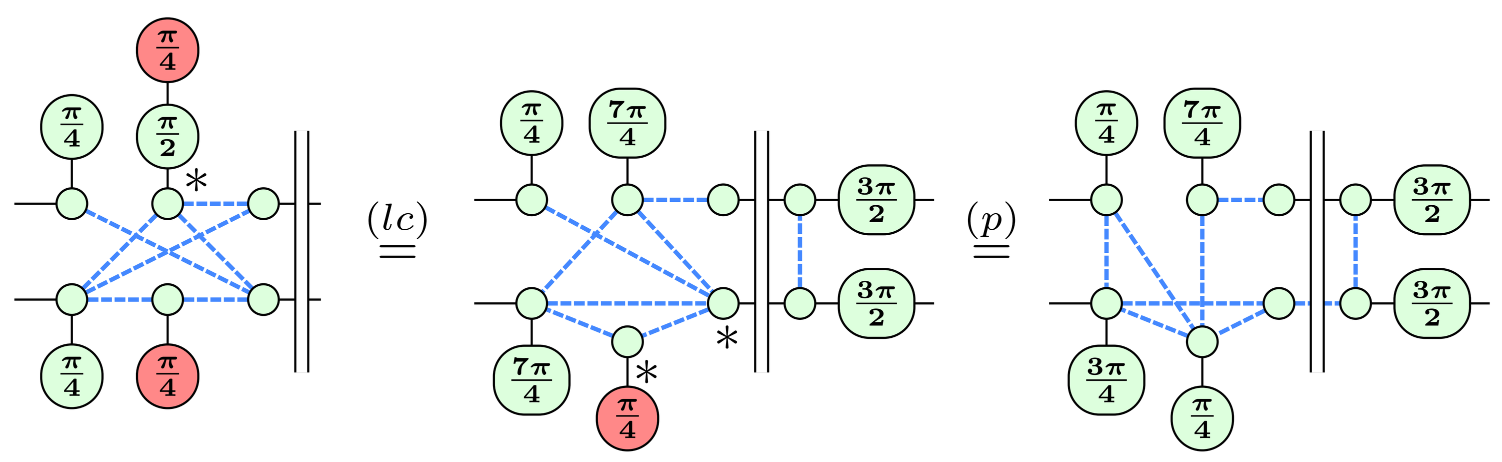
On the right is a valid *extended gflow* for the pattern. This means, we can deterministically implement the pattern by measuring the QuBits according to the partial order induced by $\prec$ and correcting unwanted measurement results on future measurements according to g.

Circuit extraction from (extended) gFlow patterns

For extracting an ancilla-free circuit from an MBQC pattern, i.e. a circuit where the number of QuBits is only as large as the number of outputs in the pattern, Backens et al. [1] present an algorithm which works for all patterns with extended gFlow. Starting from the outputs, the algorithm splits the diagram into an extracted and unextracted part and sequentially "pulls" spiders with an immediate gate counterpart on the extracted part while preserving gFlow in the unextracted part. Angles of output spiders are extracted as R_z gates, Hadamard wires between two outputs as CZ gates. If an output is only connected to a single non-output vertex, the vertex gets extracted as H gate. In cases where we cannot find such an output, we can add the edges of an output to the edges of another output by placing a CX gate on the extracted circuit:



If all vertices of the pattern are measured in XY plane, gFlow ensures, that there is always an output neighbor in the odd neighborhood of only outputs. By adding all those outputs together with CX gates, we end up with an output only connected to a single unextracted vertex. We find such a combination implicitly by gaussian elimination on the adjacency matrix; calculating an actual gflow is not necessary. XZ and YZ measurements are translated to XY measurements during the procedure using graph-theoretic transformations on the pattern. XZ measurements can be transformed via local complementation and YZ measurements by applying a Pivot on the measurement and an adjacent output:



Pauli flow extraction

A first algorithm for also extracting patterns with Pauli flow was introduced by Simmons [3]. In contrast to the gFlow extraction, this algorithm requires to actually calculate the Pauli flow of the pattern. Another way to extract patterns with Pauli flow is to use the *ZX-simplify* algorithm from [2] which brings a diagram in "reduced gadget form". Their algorithm removes every Pauli measurement in a graph-like diagram, and since a Pauli flow without such measurements is also a gFlow, we can extract the resulting diagram with the known algorithm. From the MBQC perspective we obtain a diagram in reduced gadget form by transforming all X or Y Pauli measurements to Z using local complementation/pivot and remove the Z measurements from the diagram [a]:

$\lambda(w)$	$\lambda'(w)$ after $G * u$		$\lambda'(w)$ after $G \wedge (u,v)$	
	$w = u$	$w \in N(u)$	$w \in \{u, v\}$	$w \in N(u) \cup N(v)$
X	X	Y	Z	X
Y	Z	X	Y	Y
Z	Y	Z	X	Z

However, we do not need to eliminate *every* Pauli measurement. Just like for the gFlow extraction algorithm, we can start at the outputs and pull spiders on the extracted part with the following modifications:

- Remove every Z measurement at the beginning.
- If we cannot proceed, because we do not find a combination of row additions we apply one of the following steps in this order:

1. Remove a Y output neighbor using local complementation.
2. Remove an X output neighbor with a Pivot on vertex + output.
3. Remove an X vertex adjacent to an output neighbor by applying a Pivot on vertex + output neighbor.

1. and 2. are very similar to the XZ and YZ transformations from the gFlow extraction. If we do not find a combination of row additions these steps successively remove every output neighbor not measured in XY. The third step is necessary because even if all output neighbor are measured in XY, there could be no output neighbor which has only outputs in its correction set. This is because in Pauli flow correctors can also occur in the past of a vertex. However, we know that removing every Pauli measurement would give us a gFlow and the only possible way to change the connectivity between outputs and output neighbors would be if an output neighbor is part of a Pivot to remove an adjacent X vertex.

[a] Note, that we cannot transform inputs or outputs being measured in X or Y. However, inputs and outputs can always be relabeled as XY measurements without loosing flow property.

Outlook

Since our algorithm is dominated by gaussian elimination, it has roughly the same runtime as the gFlow extraction. Compared to [2] or [3], it is less useful for tasks like quantum circuit optimization via graph-like ZX-diagrams, because the repeated application of local complementation and pivoting during extraction usually increases gate counts. A possible application for our algorithm is to help identifying which Pauli measurements in a pattern can be relabeled as planar measurements: All vertices which we do not remove in steps 1.-3. can be relabeled as XY measurements.

References

[1] Miriam Backens, Hector Miller-Bakewell, Giovanni de Felice, Leo Lobski, and John van de Wetering. There and back again: A circuit extraction tale. *Quantum*, 5:421, 2021.

[2] Aleks Kissinger and John van de Wetering. Reducing the number of non-clifford gates in quantum circuits. *Physical Review A*, 102(2):022406, 2020.

[3] Will Simmons. Relating measurement patterns to circuits via pauli flow. *arXiv preprint arXiv:2109.05654*, 2021.