Every non-signalling channel is common-cause realizable

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In this work, we investigate the relation between the two options.

Outline

1. Preliminaries
   - Generalized probabilistic theories (GPTs)
   - Non-signalling channels
   - Common-cause realizations
2. Setting up the problem
   - Common-cause completions
3. Our construction
   - Overview
4. Final remarks

Generalized probabilistic theories

(focus on compositionality)

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*Causal, locally-tomographic

Compositional Structure

Symmetric monoidal category: **diagrams**

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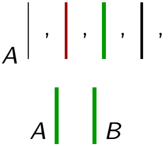

Processes:

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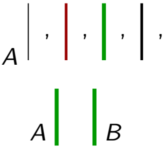

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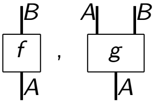

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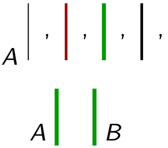

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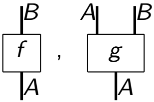

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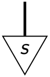

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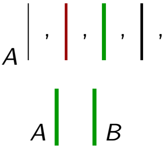

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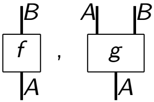

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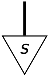

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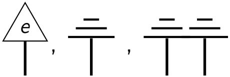

Convex structure:

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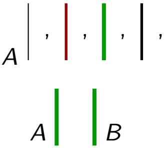

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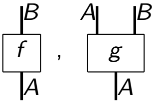

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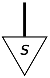

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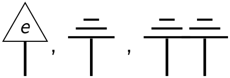

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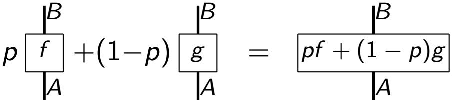

Compositional Structure

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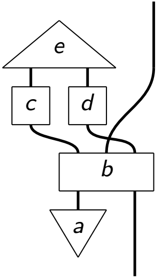

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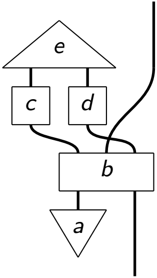

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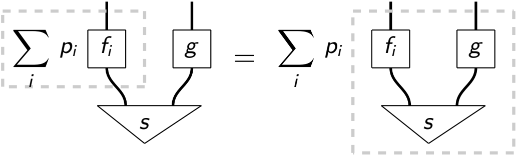

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Real vector spaces, stochastic maps, tensor and matrix products:

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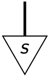

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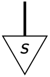

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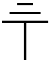 $= \begin{pmatrix} 1 & 1 \end{pmatrix}$

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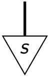

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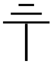

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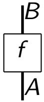

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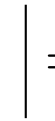
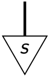
 $= \mathbb{R}^2$



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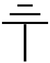
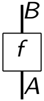
 $= \begin{pmatrix} 1 & 1 \end{pmatrix}$



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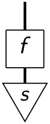

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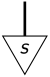

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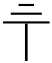
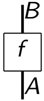
 $= \begin{pmatrix} 1 & 1 \end{pmatrix}$



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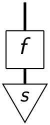

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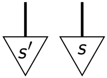

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With the composition rules in mind:

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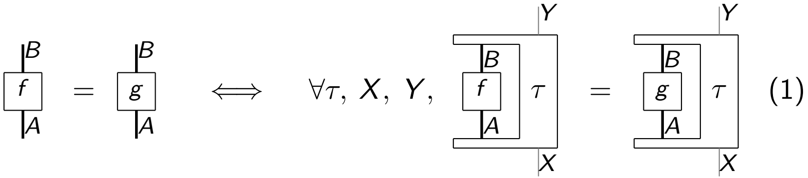
Let's discuss the last two points

Equality of processes

Tomography



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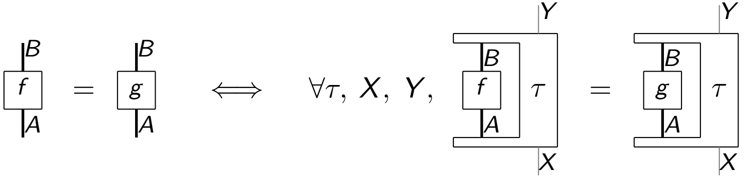
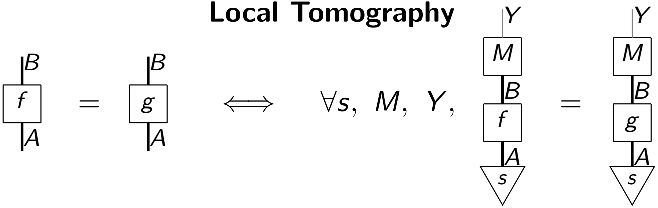


(1)

Local Tomography



(2)

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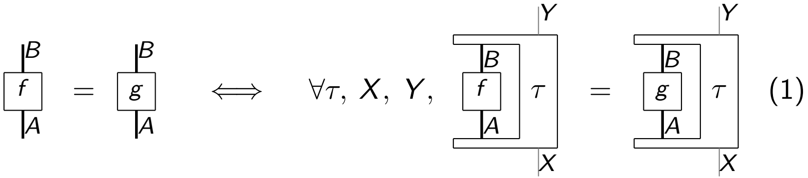

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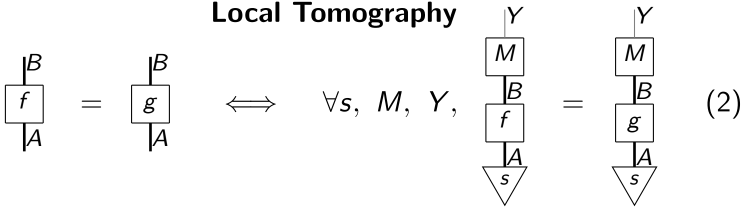
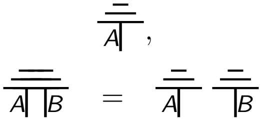


***Equality by operational equivalence**

Causality

Unique discarding effect



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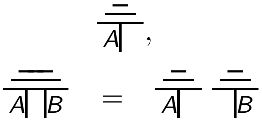

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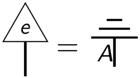

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Causality

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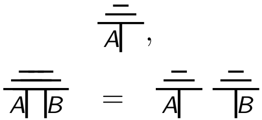

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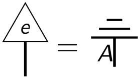

(4)

Our probabilities come from inside the stochastic maps and state vectors

With the context of GPTs in mind, we can now define the notions of **common-cause realisation** and **non-signalling**.

Non-signalling channels

Intuition

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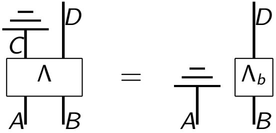
Non-signalling from AC to BD:



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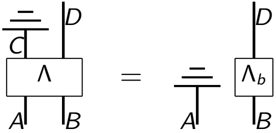

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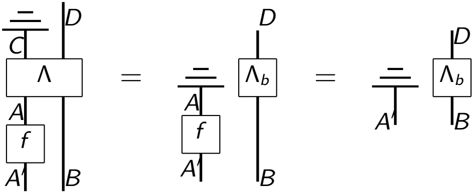

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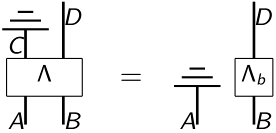

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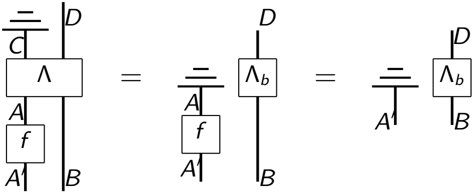
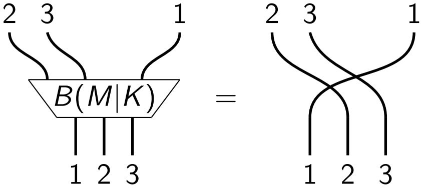


(6)

A channel is NS if it can't signal between any two parties.

Definition: Multipartite case

Consider $B(M|K)$ are bipartitions of wires such as



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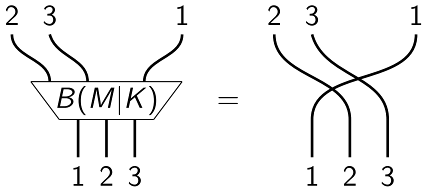

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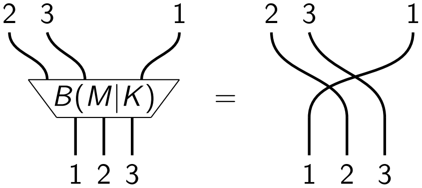

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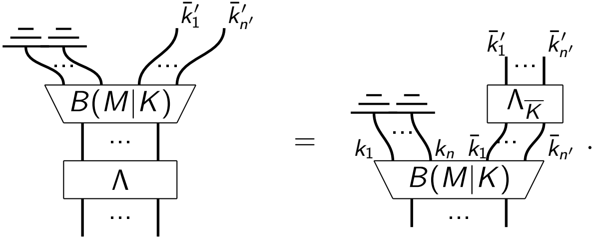
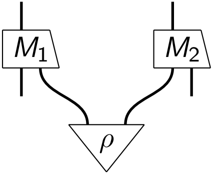


(8)

Common-cause realization

Prototypical example

Bell scenario



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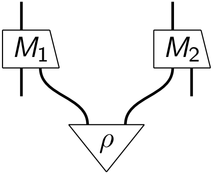
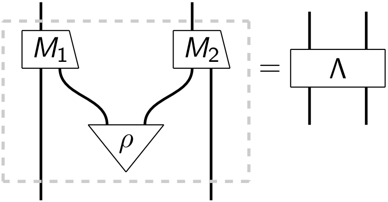


(9)

The state s can be seen as a **common-cause** for the two measurements

Prototypical example

If we group the pieces,



(10)

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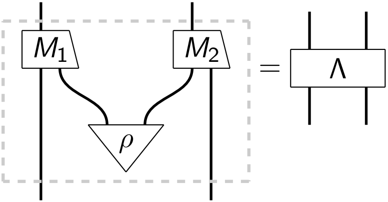
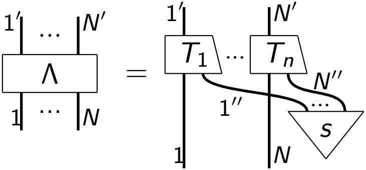


(10)

Then ρ, M_1, M_2 provide a **common-cause decomposition** of Λ.

Definition

Common-cause decomposition



(11)

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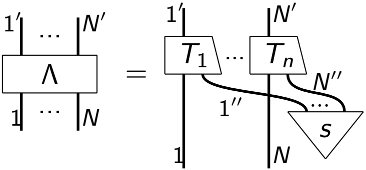

(11)

Is a common-cause decomposition of Λ.
Note that this implies Λ is non-signalling.

If we don't have a common-cause decomposition, we can still ask whether it exists within **some** other theory.

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GPT-common-cause realisable channel

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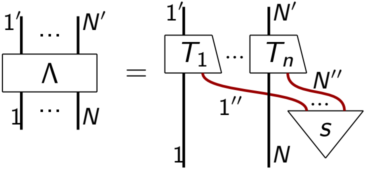
$\Lambda \in \mathbf{G}$ is GPT-common-cause realisable iff there exists a theory such that



(12)

and the original theory $\mathbf{G}$ is a **full subtheory** of the new one.

Interlude

What to keep in mind:

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4. We diagramatically define non-signalling with the discarding effect.
5. We diagramatically define common-cause realisations with shared states.

The problem

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We know that:

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Boxworld is a **common-cause completion** of classical theory.

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So, we **generalise that observation** to formulate our problem.

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Can we always find a common-cause completion of a causal locally tomographic GPT?

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Yes

Common-cause completions

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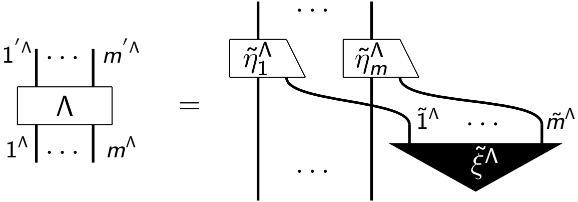
Let's look at the basic idea of the construction.

Common-cause completions

A previous result[1], guarantees we can always write



(13)

Where $\tilde{\xi}^{\Lambda}$ is an (unphysical) affine combination of states from the GPT.

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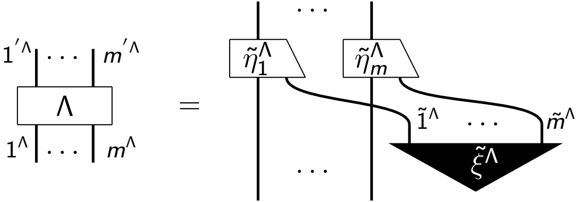

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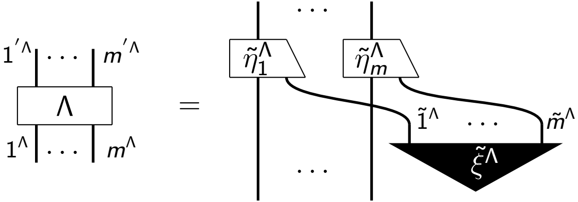

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Why? Negative probabilities

[1] P. J. Cavalcanti, J. H. Selby, J. Sikora, and A. B. Sainz, Journal of Physics A: Mathematical and Theoretical (2022)

If we can find a way to add processes $\tilde{\xi}^{\Lambda}, \tilde{\eta}_i^{\Lambda}$ to the GPT in a consistent way, then we can use that theorem to construct a common-cause completion.

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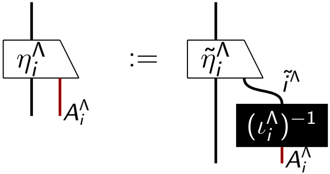
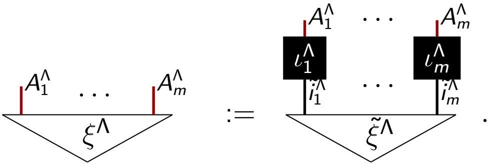
So, **let's instead add**



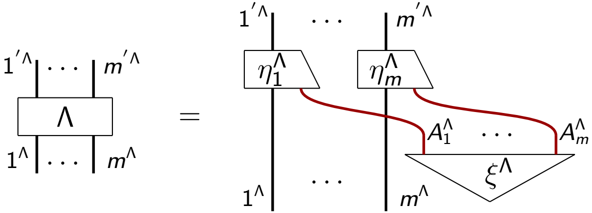
(14)

and



(15)

Now, we can rewrite the theorem decomposition as



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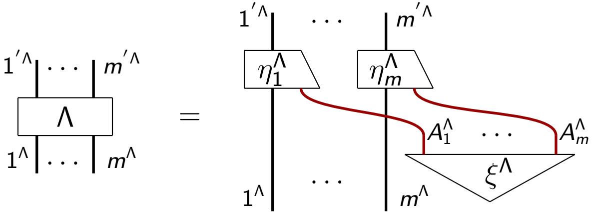

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And we can't freely compose ξ^{Λ} with other processes without going through η_i^{Λ} first

This is the key idea of the construction.

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That leads us to our main result

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Theorem V.1. Given a locally-tomographic causal GPT **G**, its set of multipartite non-signalling channels is the same as its set of multipartite common-cause realisable channels.

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Note these common-causes might not be state-preparations allowed in **G**.

Outlook

- We investigate the relation between the **non-signalling** channels and the **GPT-common-cause realisable** channels of **causal, locally tomographic GPTs**.

[2] D. Schmid, H. Du, M. Mudassar, G. Coulter-de Wit, D. Rosset, and M. J. Hoban, Quantum 5, 419 (2021)

[3] P. J. Cavalcanti, J. H. Selby, J. Sikora, T. D. Galley, and A. B. Sainz, npj Quantum Information 8, 1 (2022)

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- Our result provides a more principled reason to use the non-signalling channels as the enveloping theory in resource theories.

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Open Questions

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2. Can we find a similar scheme for when $\mathbf{G}$ is not locally tomographic?

The End

Thank you!

Take home message:

- **Non-signalling** channels coincide with **GPT-common-cause realisable** channels in **causal, locally tomographic GPTs**.
- This provides a causal justification for using non-signalling channels as the enveloping theory in resource theories.

arXiv:2307.03489