Classically Simulating Quantum Supremacy IQP Circuits through a Random Graph Approach

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Overview

What's an IQP Circuit?

An Instantaneous Quantum Polynomial (IQP for short) can be represented by

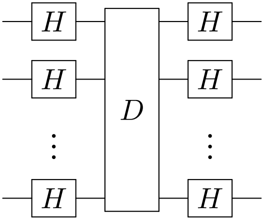
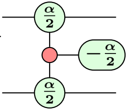


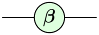
Figure: Shape of a generic IQP circuit

Where D is made out of diagonal 2 qubits gates

What's an IQP Circuit?

Precisely, D is made out of

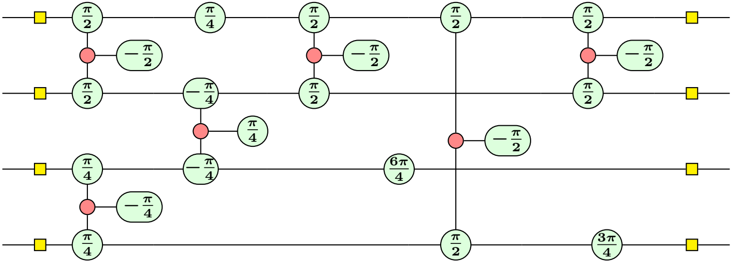
$\mathrm{CZ}_\alpha = \sqrt{2}$ 

$Z_\alpha =$ 

In the Clifford+T fragment, $\alpha = k\frac{\pi}{2}$ and $\beta = k\frac{\pi}{4}$ for some $k \in \{0, 1, 2, ..., 7\}$

Simplified Form

example



Simplified Form

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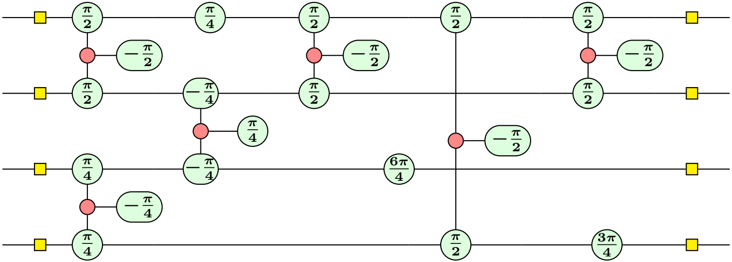
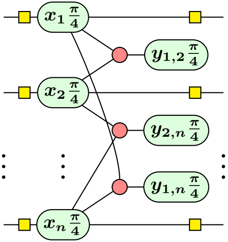
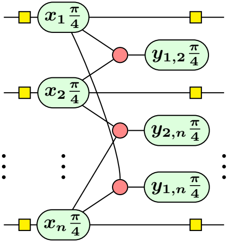




Random IQP Families

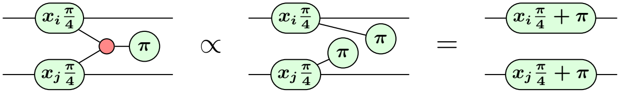
Dense $\rightarrow$ Pick $x_i, y_{i,j}$ uniformly at random i.i.d $\forall i, j$



Sparse $\rightarrow$ same but every pair interact with probability $\lambda \frac{ln(n)}{n}$

Usefull Fact

When a phase gadget has a phase of 0 or π



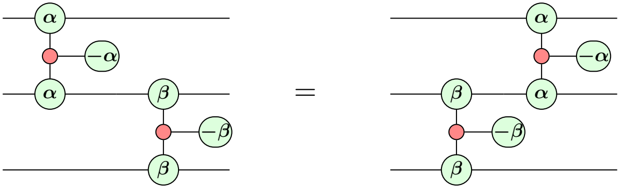
There is no gadget

Why Do We Care?

- "Easy" to implement on a NISQ computer
- Hard to classically simulate
- Supremacy experiments?

Runnable on NISQ Machine?

All the gates commute



No time ordering $\Rightarrow$ less noise

The sparse family can be compiled into a 2D lattice of depth $O(\sqrt{n}\log(n))$

Resilient to noise

Runnable on NISQ Machine?

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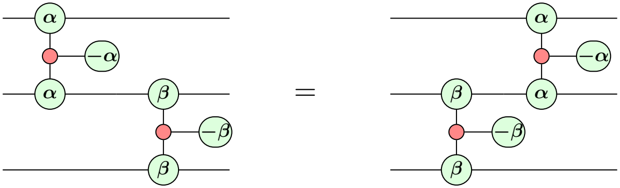

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Resilient to noise

Hard to classically Simulate

- Efficient weak simulation (up to a $\sqrt{2}$ multiplicative error) implies the collapse of PH
- Average case is hard to simulate within a small additive error[1] (implies the collapse of PH)
- even the sparse case is hard to simulate[2]

[1] Assuming some hardness conjecture about random polynomial over $\mathbb{F}_2$

[2] Assuming some conjecture about the ising model

Quantum Supremacy?

Good properties for quantum supremacy[3]
How good is the classical simulation?

	Dense	Sparse
Tensor contraction[4]	$O(n2^n)$	$O(\log n2^n)$
Stabiliser decomposition	$O(n^3 2^{O(n^2)})$	$O(n \log^2 n 2^{O(n \log n)})$

[3] Or quantum inimitably
[4] Used state vector to get the bound

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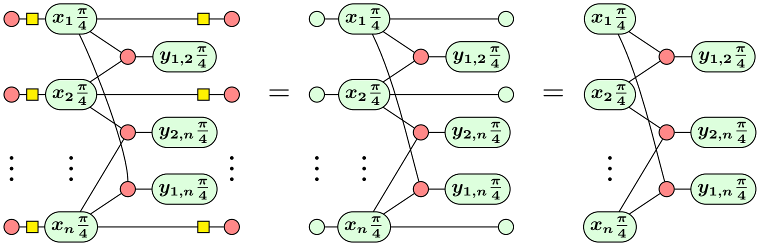
Ours (improved)	$O\left(\frac{(\log n)^{4-\alpha}}{n^{2-\alpha}} 2^n\right)$	-

Where $\alpha \approx 0.396$

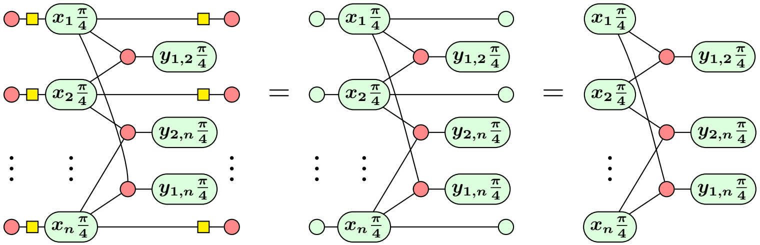
[3] Or quantum inimitably
[4] Using state vector

Strong Simulation



Note: IQPs have an efficient strong simulation $\rightarrow$ weak simulation reduction

Strong Simulation



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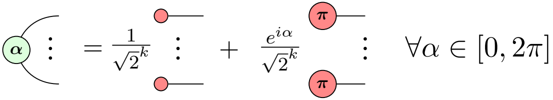
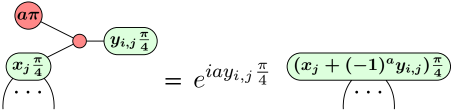
Cutting a Qubit

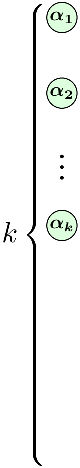




If we do this on all the qubits $\rightarrow O(n2^n)$

Idea

We do this until we are left with k disconnected spiders



We get $O(2^{n-k})$ terms Goal: Maximize k

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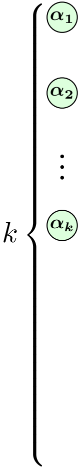

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Interaction Graph

A vertex per qubit
An edge between two qubits if they interact non trivially

The maximum k is the size of the maximal independent set

It's a random graph under the Erdős–Rényi model

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Erdős–Rényi model

Every edge is there with some (independent) probability p.

Dense $\rightarrow G(n, \frac{3}{4})$

Sparse $\rightarrow G(n, \frac{3\gamma \ln(n)}{4n})$

Independent Set in Random Graphs

For dense graphs:

For sparse graphs (in our regime):

Independent Set in Random Graphs

For dense graphs:

Theorem (Matula, 1972)

For $p \in (0, 1)$*,* $\alpha(G(n, p))$ *is tightly concentrated around* $2 \log_{1/(1-p)} n$

For sparse graphs (in our regime):

Independent Set in Random Graphs

For dense graphs:

Corollary

Let $p \in (0,1)$, $b = \frac{1}{1-p}$ *then* $\alpha(G(n,p)) \geq 2\log_b n - 2\log_b(\log_b(n))$ *with high probability.*

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$\implies k \approx \log_2 n - \log_2 \log_2 n$

For sparse graphs (in our regime):

Independent Set in Random Graphs

For dense graphs: $k \approx \log_2 n - \log_2 \log_2 n \implies O\left(\frac{\log^2 n}{n} 2^n\right)$

For sparse graphs (in our regime):

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For sparse graphs (in our regime):
No direct theorem, but with a bit more work but we can prove

Theorem

There exists a constant $C > 0$ such that with high probability

$$\alpha\left(G\left(n, \frac{3\gamma \ln(n)}{4n}\right)\right) \geq C\frac{n \log\log(n)}{\log(n)}.$$

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Theorem

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$$\alpha\left(G\left(n, \frac{3\gamma \ln(n)}{4n}\right)\right) \geq C\frac{n \log\log(n)}{\log(n)}.$$

$$\implies O\left(\frac{n \log\log(n)}{\log(n)} 2^{n\left(1 - \frac{C \log\log(n)}{\log(n)}\right)}\right)$$

Independent Set in Random Graphs

For dense graphs: $k \approx \log_2 n - \log_2 \log_2 n \implies O\left(\frac{\log^2 n}{n} 2^n\right)$

For sparse graphs (in our regime):

$$O\left(\frac{n \log\log(n)}{\log(n)} 2^{n\left(1 - \frac{C \log\log(n)}{\log(n)}\right)}\right)$$

(Which is faster than $O(2^n / poly(n))$)

Why Did it Work?

- Instead of decomposing all the $O(n^2)$ T-gates, we got away with $O(n)$
- We stopped early when we had an easy diagram left
- Offloaded the analysis to random graphs

Could we have stopped earlier?

Why Did it Work?

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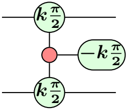
Non-Clifford Interaction Graph

Same a before but we only have edges between two qubits if they interact by a non-Clifford phase gadget

Recall, we have



Dense $\rightarrow G(n, \frac{1}{2})$
Sparse $\rightarrow G(n, \frac{\gamma \ln(n)}{2n})$

Finish with general Stabiliser decomposition

Left with a non trivial ZX-diagram with $O(n)$ T-gates.

We use general stabiliser decomposition

Dense $\rightarrow O\left(\frac{(\log n)^{4-\alpha}}{n^{2-\alpha}} 2^n\right)$

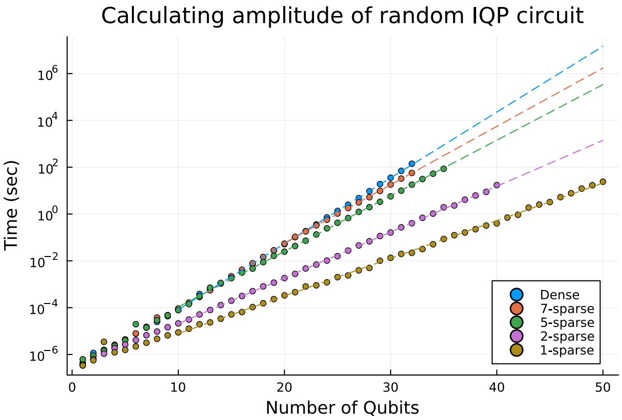
Benchmarks

Basic algorithm implemented in Rust
`https://github.com/Codsilla/iqp-sim-independent-set`

Single-threaded on a consumer laptop (Intel Core i7-10750H CPU 2.60GHz)

Based on the average of 100 instances

Benchmarks



It fits remarkably well to an exponential fit $c2^{\beta n}$ where β ranges from 0.93 to 0.53.

Take Away / Conclusion

- Polynomial speedups
- Works well in practice
- Highly parallelizable

Are IQPs really a good idea for supremacy?

Extensions

Looking at other graphs properties

Ex: Maximal induced planar subgraph guarantees good network splicing. It can be evaluated in $\tilde{O}(2^{\sqrt{n}})$

Thank you for your attention!

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