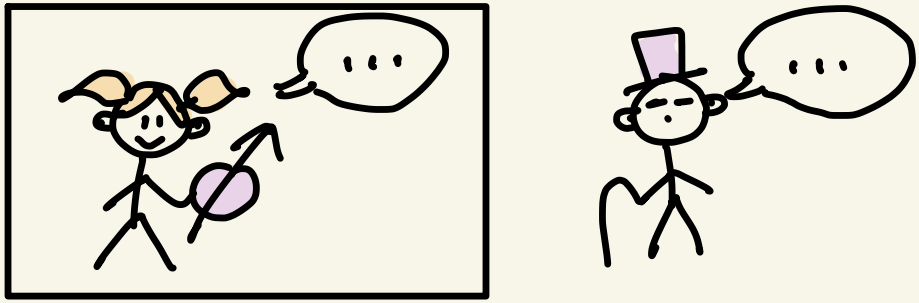
Multi-agent paradoxes & contextuality

by Nuriya Nurgalieva

ETH Zürich



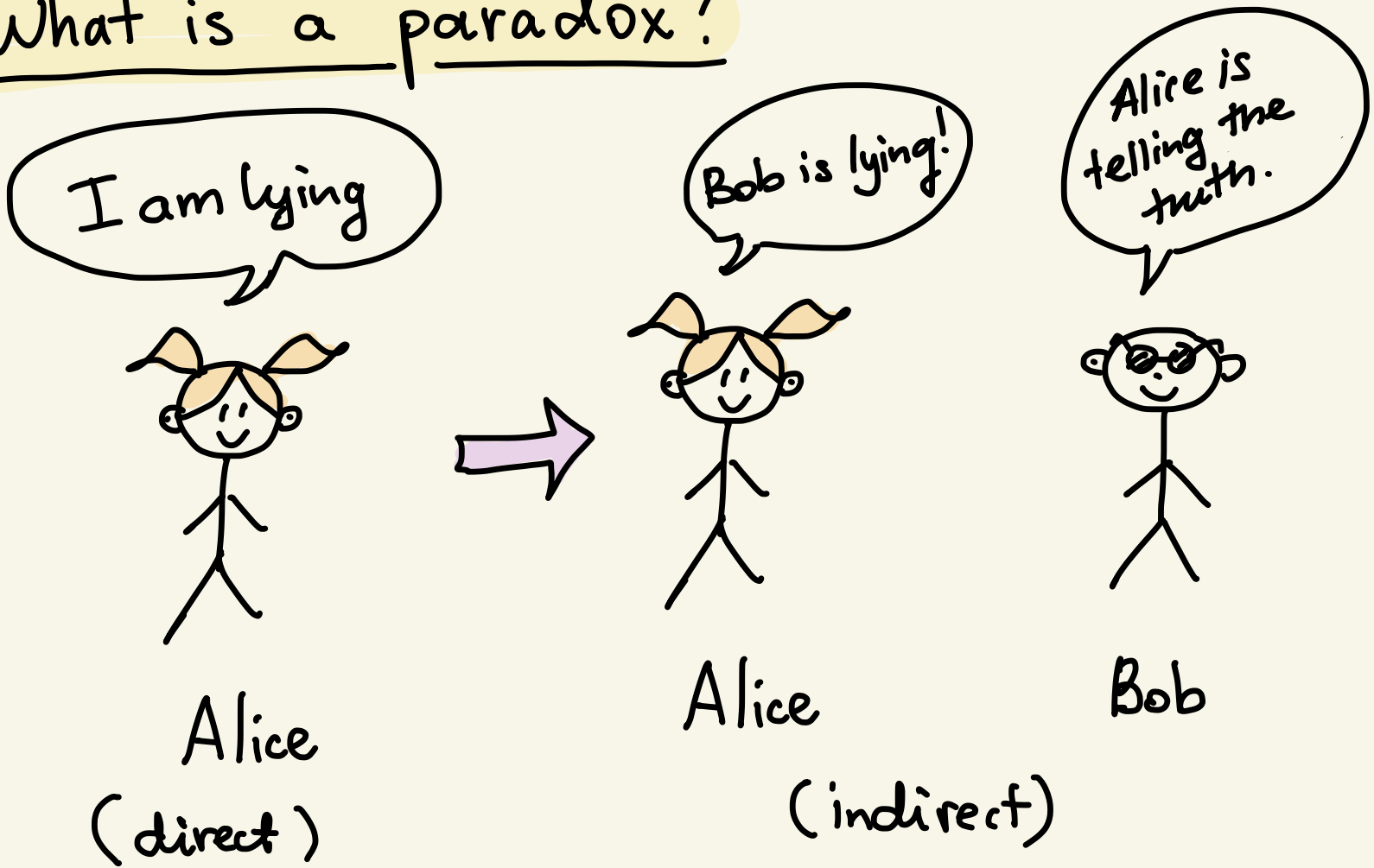
based on joint work with V. Vilasini

QPL 2023

Contents

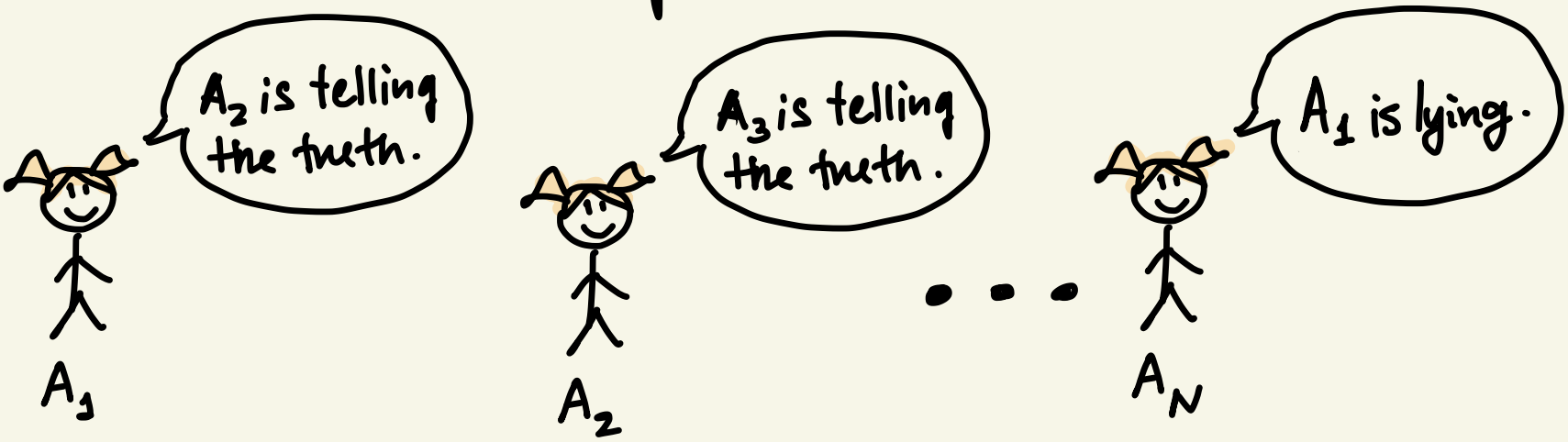
* what is a paradox?
* structure of epistemic paradoxes
* logical contextuality
* multi-agent paradoxes in phys. theories
* main theorem

What is a paradox?



Liars cycle

An extension to N parties



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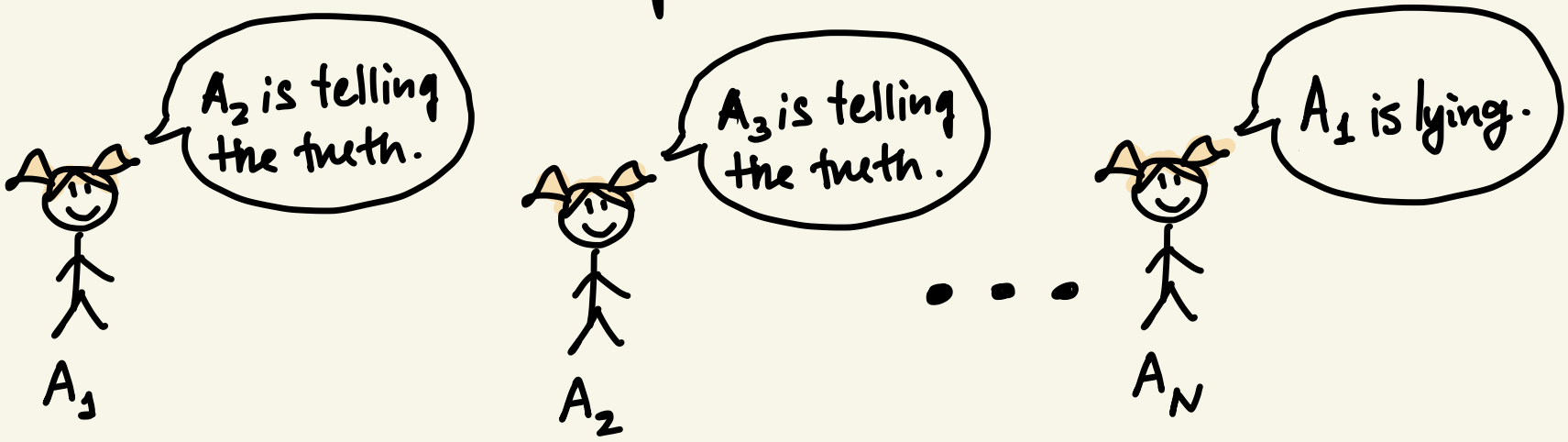

$a_1 = 1 \Leftrightarrow a_2 = 1$ $\quad a_2 = 1 \Leftrightarrow a_3 = 1$ $\quad \dots$ $\quad a_N = 1 \Leftrightarrow a_1 = 0$

Structure of epistemic paradoxes

Reference relation graphs

set of prop.

$G_g = (\Sigma, E)$

edges

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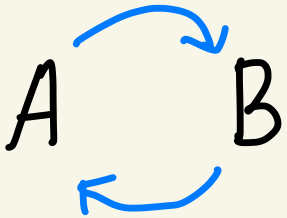

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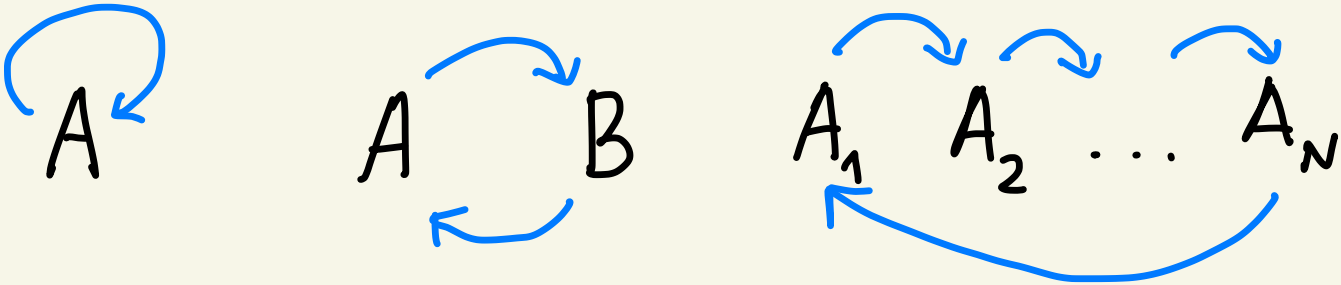

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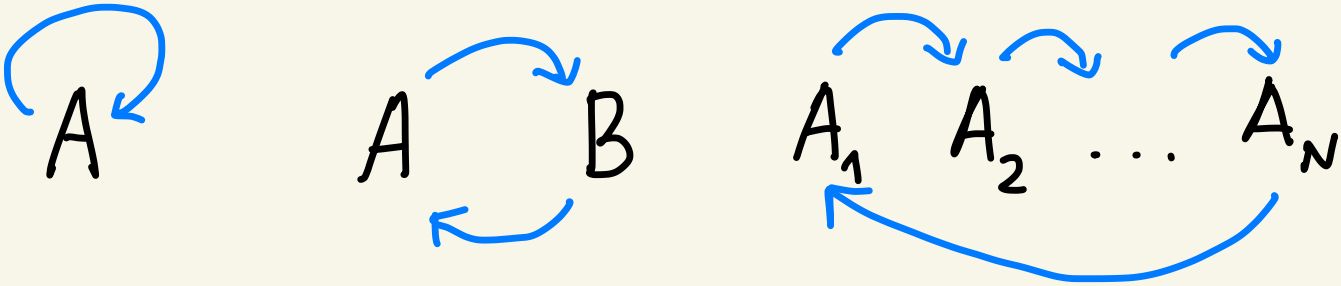
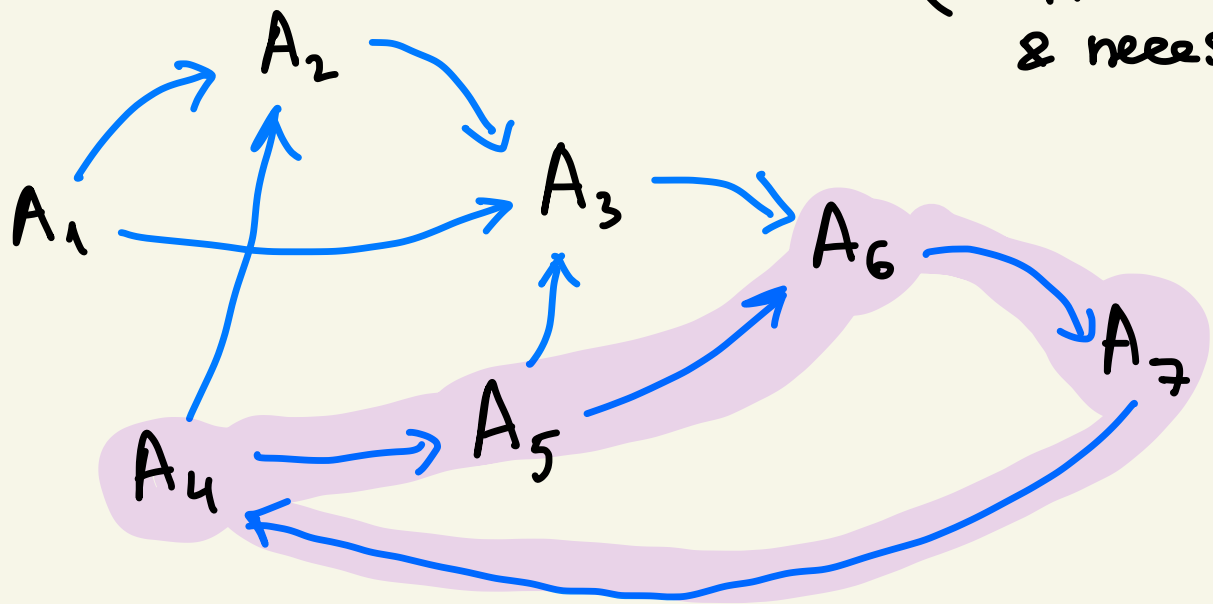


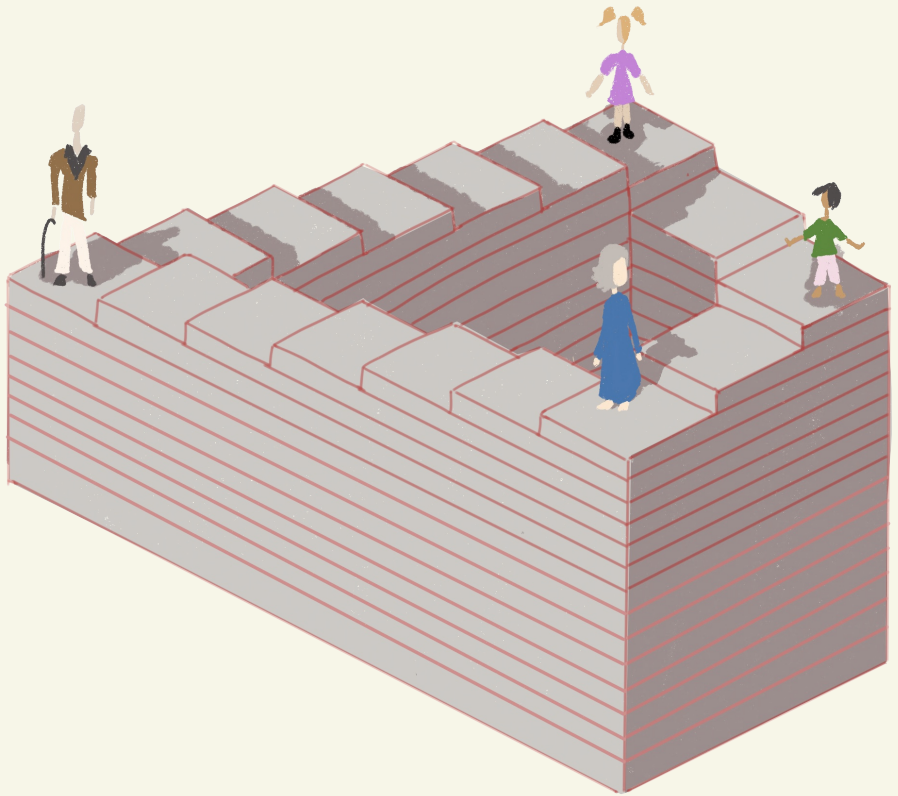
Remark: ∃ infinite chain paradoxes
(e.g. Yablo's paradox)

Structure of epistemic paradoxes

Rabern et al : directed cycle or double path (sufficient & necessary)



Contextuality



locally consistent

BUT

globally inconsistent

Contextuality

Measurement scenario: $(X, \mathcal{U}, O)$

- X: Set of variables
- $\mathcal{U}$: family of subsets of X
- O: set of outcomes

Obtaining outcomes

$$s \cdot C^{\in \mathcal{U}} \rightarrow O^{C \subseteq O}$$

Contextuality

Set of all possible outcome assignments $S(C)$

Compatible family: $\{s_C\}_{C \in \mathcal{U}}$ $(s_C \in S(C))$

$$s_C|_{C \cap C'} = s_{C'}|_{C \cap C'}$$

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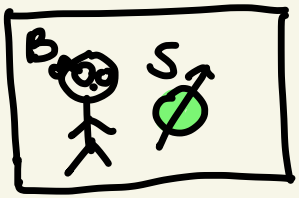
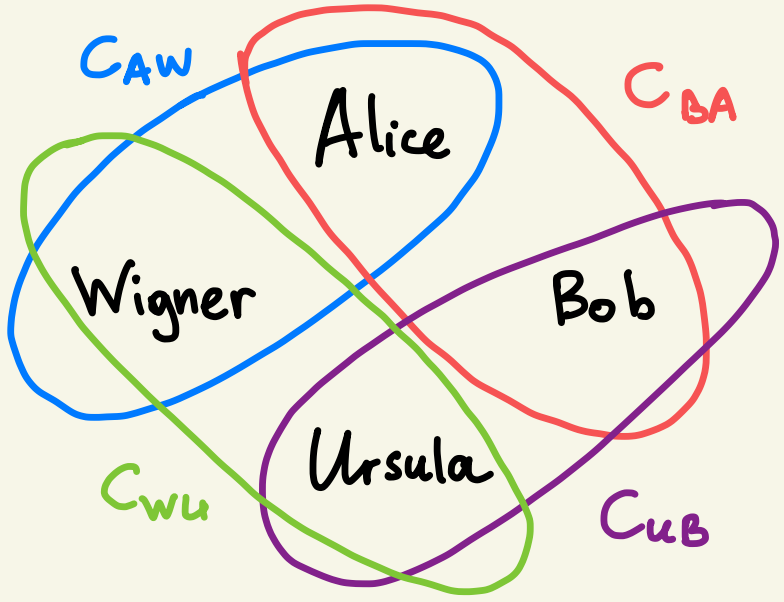
Contexts in Frauchiger-Renner scenario

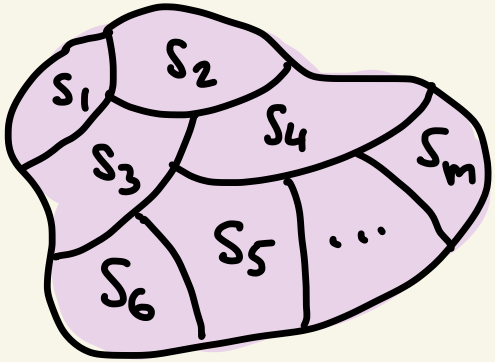




Multi-agent paradoxes in physical theories

Includes: agents, systems, memories, measurements, ...

Important: we only consider reducible scenarios



$\rho_{s_1 \dots s_m}$

Conclusions can be made based on this state alone

Multi-agent paradoxes in physical theories

Few more ingredients:

* common theory
* compatibility of agents & trust
* distribution of knowledge
* setting-independence
* non-contradictory single outcomes

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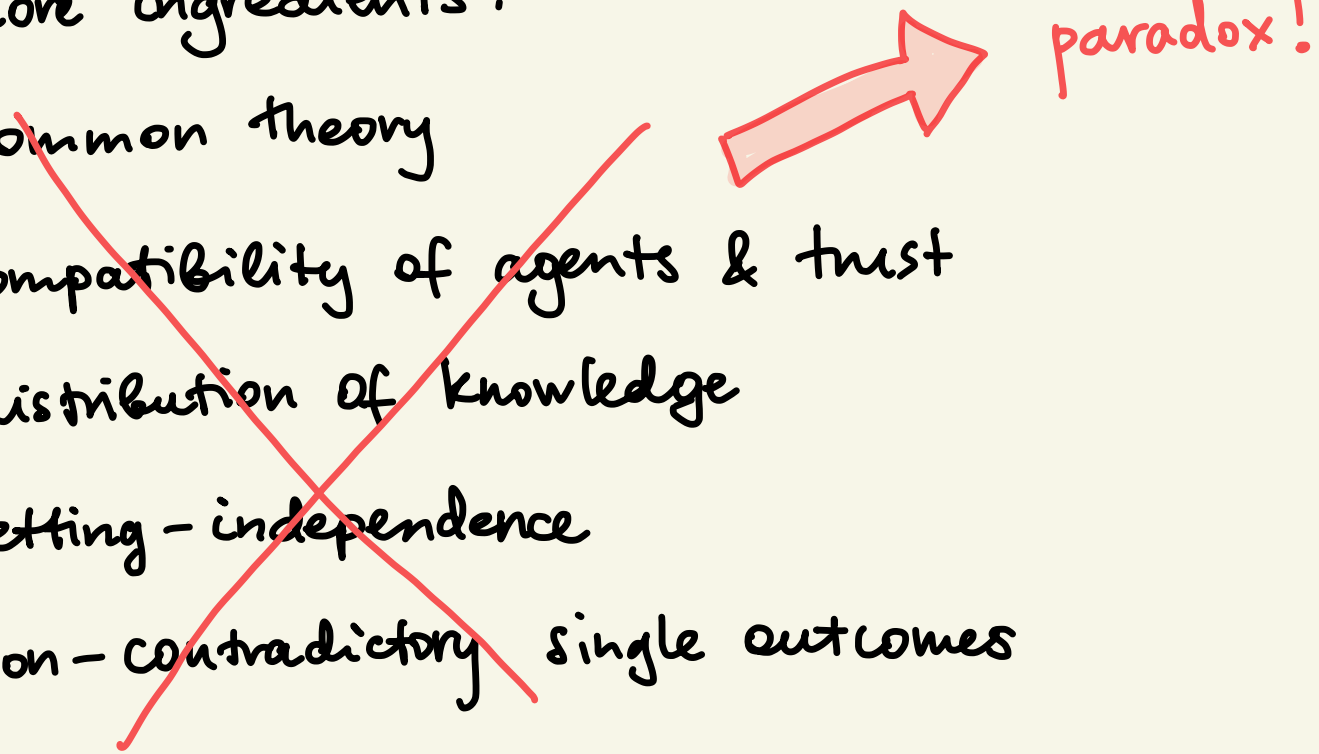

Main theorems

(1) Reducible multi-agent paradoxes in a theory T are proof of logical contextuality of the theory.

(2) Every reducible multi-agent paradox can be simplified to a Liars cycle-type chain of statements.

Outlook

* drop the reducibility assumption

* towards a unified view on no-go theorems

Results I didn't mention

* mapping KCBS contextuality scenarios to multi-agent paradoxes

* Some properties of paradoxes in QT

Thank you for your attention!

→ paper to soon appear on arXiv
(also see my thesis)