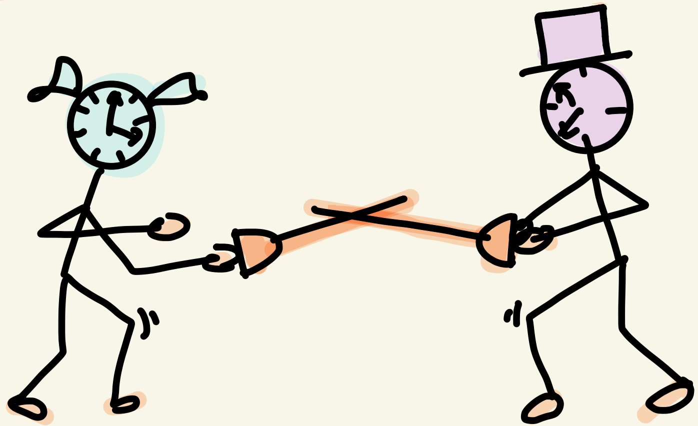
Reconstructing the waiting time of the clock using the simplest reference

(aka putting clocks against each other)



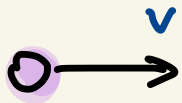
QPL 2023

by Nuriya Nurgalieva

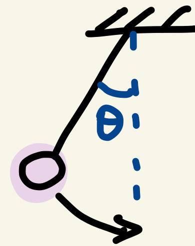
based on joint work with Ralph Silva and Renato Renner

Contents

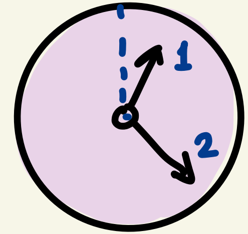
Time: measured indirectly via clocks



$$t = \frac{x_1 - x_0}{v}$$

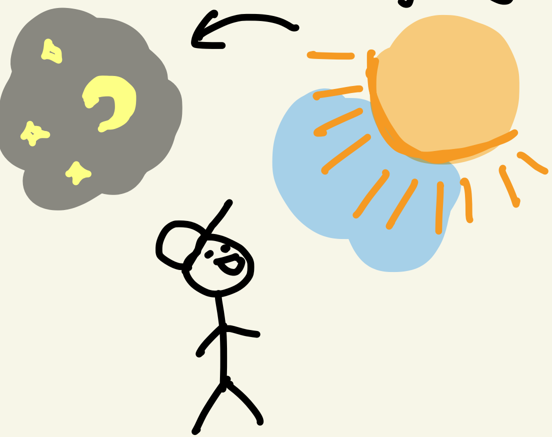


$$t = f(\theta)$$



$$t = F(\theta_1, \theta_2)$$

CLOCK : any system that is measured to infer time

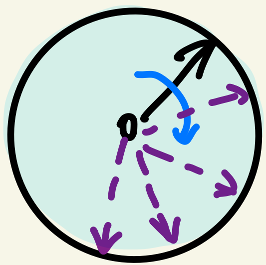


any interaction that correlates deg. of freedom of clock with environment

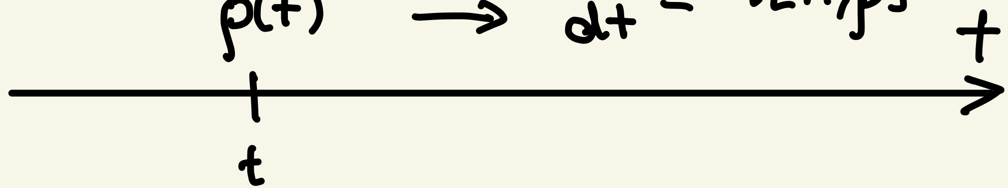
What is a clock?

„clock": it ticks!

we assume background time



$$\rho(t) \longrightarrow \frac{d\rho}{dt} = -i[H,\rho]$$



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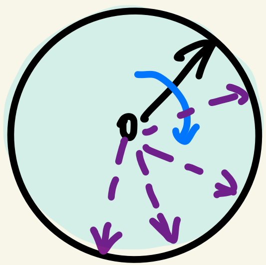

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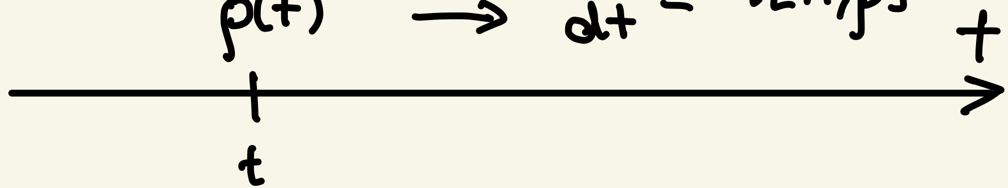
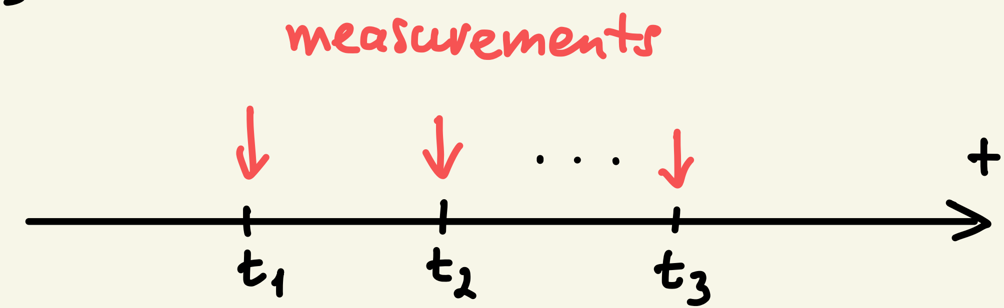


clock: measurements of time



Dynamics & structure of a clock

$\rho = \sum_k A_k \rho A_k^{\dagger}$ ⟶ Koashi-Imoto theorem

$$\mathcal{H} = \bigoplus_n \mathcal{H}_{C_n} \otimes \mathcal{H}_{F_n}$$

register

clockwork

display

add. assumptions ⟶ a simpler model

see arXiv. 230601829

Dynamics & structure of a clock

$$\rho_{CT} = \sum_n \rho_C^{(n)} \otimes |n\rangle\langle n|_T$$

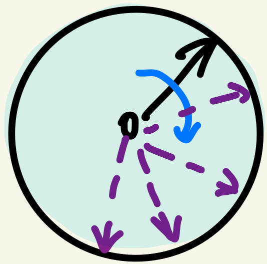
internal clockwork

classical register
(gives us the # of ticks when meas.)

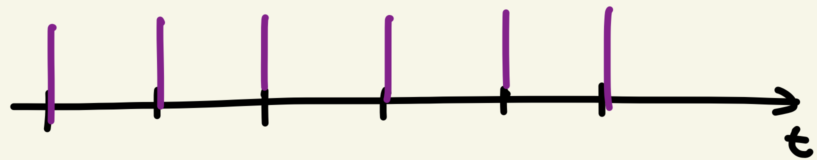
Lindbladian dynamics

(does not move the register) + (increases the register by 1)

Ideal clock vs „real" clock

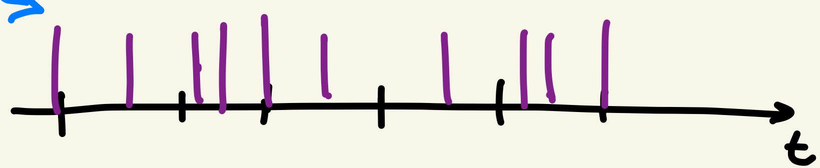


ideal clock

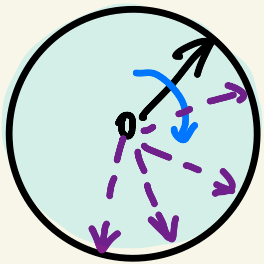


real clock

$P(T_1 = t_1, T_2 = t_2, \dots)$



Waiting time distribution



$$P(T_1 = t_1, T_2 = t_2, \ldots)$$

$$\parallel \quad \text{iid ticks}$$

$$w(t_1) \cdot w(t_2 - t_1) \cdot \ldots$$

waiting time distribution
(delay function)

How to find out $w(t)$?

Physical clocks are well-behaved

$$\frac{d}{dt}\rho_{CT} = -i\,[H_c \otimes \mathbb{1}_T, \rho_{CT}] + (\,..\,)$$

$$+ \sum_m (J_m \otimes \Gamma_+)\, \rho_{CT}\, (J_m^+ \otimes \Gamma_-) - \frac{1}{2}\{J_m^+ J_m \otimes \Gamma_- \Gamma_+, \rho_{CT}\}$$

$$\Gamma_- = |n\rangle\langle n+1|$$

$$\Gamma_+ = |n+1\rangle\langle n|$$

information out:

$P(n|t)$

Physical clocks are well-behaved

$$\rho_c \otimes |0\rangle\langle 0|_T \longrightarrow \sum_{n=0}^{N} \rho_c^{(n)}(t) \otimes |n\rangle\langle n|_T$$

Simplified picture:

$$\text{iid ticks} \longrightarrow \{|0\rangle_T, |1\rangle_T\}$$

$$\omega(t) = -\frac{d}{dt}\mathrm{tr}\left(\rho_c^{(0)}(t)\right) = \mathrm{tr}\left(\sum_m J_m^\dagger J_m \rho_c^{(0)}(t)\right)$$

Physical clocks are well-behaved

Thm. The waiting time distribution $P(T_1 = t) = w(t)$ can be bounded by some decay envelope.

Intuition

eigenvalues

$\lambda = 0$ (steady)

$\mathrm{Re}(\lambda) < 0$ (decaying)

$$\rho_{CT}(t) = e^{\mathcal{L}t}\left(\rho_C^{(0)} \otimes |0\rangle\langle 0|_T\right)$$

all steady states (clock ticks)

$$\rho_{CT}(t) \in \underbrace{L(\mathcal{H}_C)}_{\text{not ticked}} \oplus \underbrace{L(\mathcal{H}_C)}_{\text{ticked}}$$

Physical clocks are well-behaved

Thm. The waiting time distribution $P(T_1 = t)$ can be bounded by some decay envelope.

Intuition

generalized eigenmatrices $\mathrm{Re}(\lambda_j) < 0$

$$\rho_c^{(0)}(0) \otimes |0\rangle\langle 0|_T = \sum_j c_j \, \mathrm{Poly}(\sigma_j)$$

$$\rho_c^{(0)}(t) \otimes |0\rangle\langle 0|_T = \sum_j c_j \, \mathrm{Poly}(\sigma_j) \, e^{-\lambda_j t}$$

$$|\rho_c^{(0)}(t)| \leq C \cdot e^{-\lambda_{min} t}$$

Physical clocks are well-behaved

Thm. The waiting time distribution $P(T_1 = t)$ can be bounded by some decay envelope.

$\longrightarrow$ it is well-behaved. ☺

- all its moments exist
- charasteristic function can be written as Taylor series... etc.

Poisson clocks

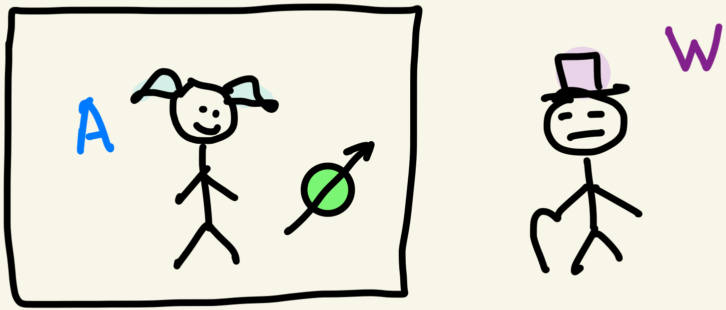
$$P(T_n - T_{n-1} = t) = \gamma e^{-\gamma t}$$

(time between ticks)

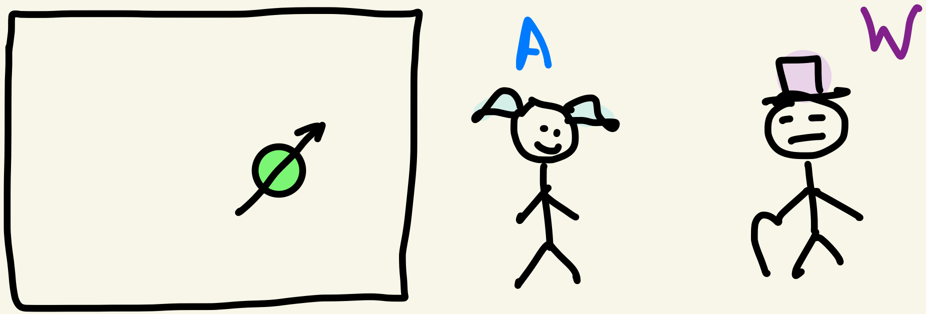
$$P(N = n \mid t) = \frac{(\gamma t)^n e^{-\gamma t}}{n!}$$

* memoryless
* $R = \frac{\mu^2}{\sigma^2} = 1$ (measure of precision)
* arises from
  - $d = 1$ for clockwork
  - $\Delta S = 0$ (equilibrium)

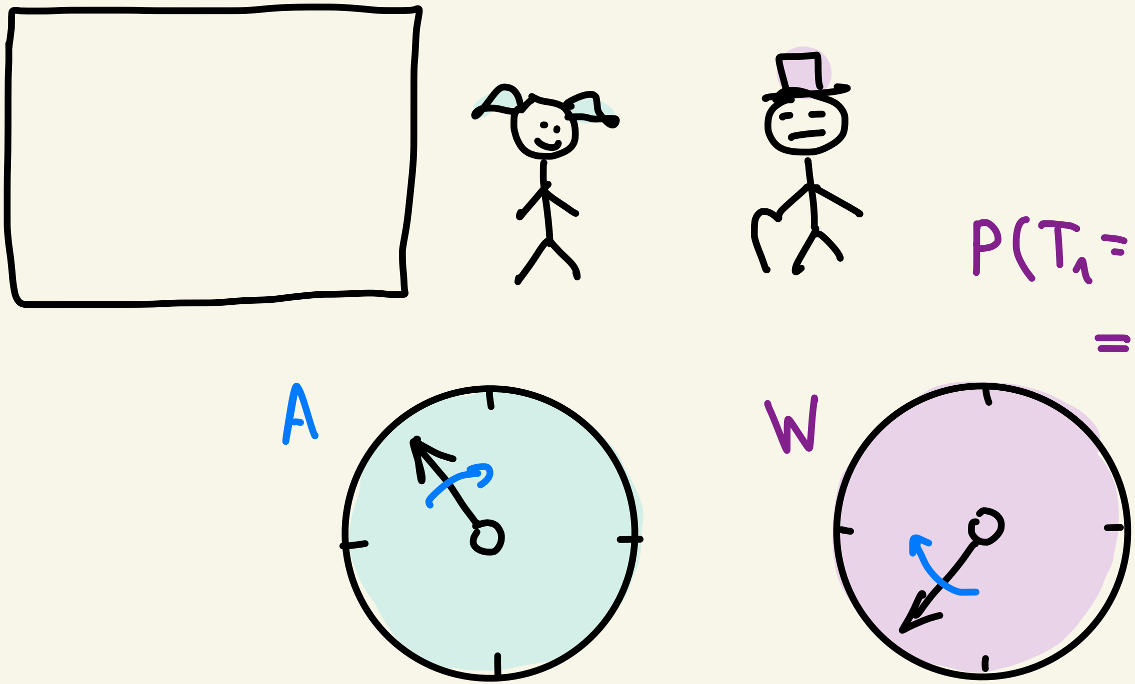
Arbitrary clock vs. Poisson clock



Arbitrary clock vs. Poisson clock



Arbitrary clock vs. Poisson clock

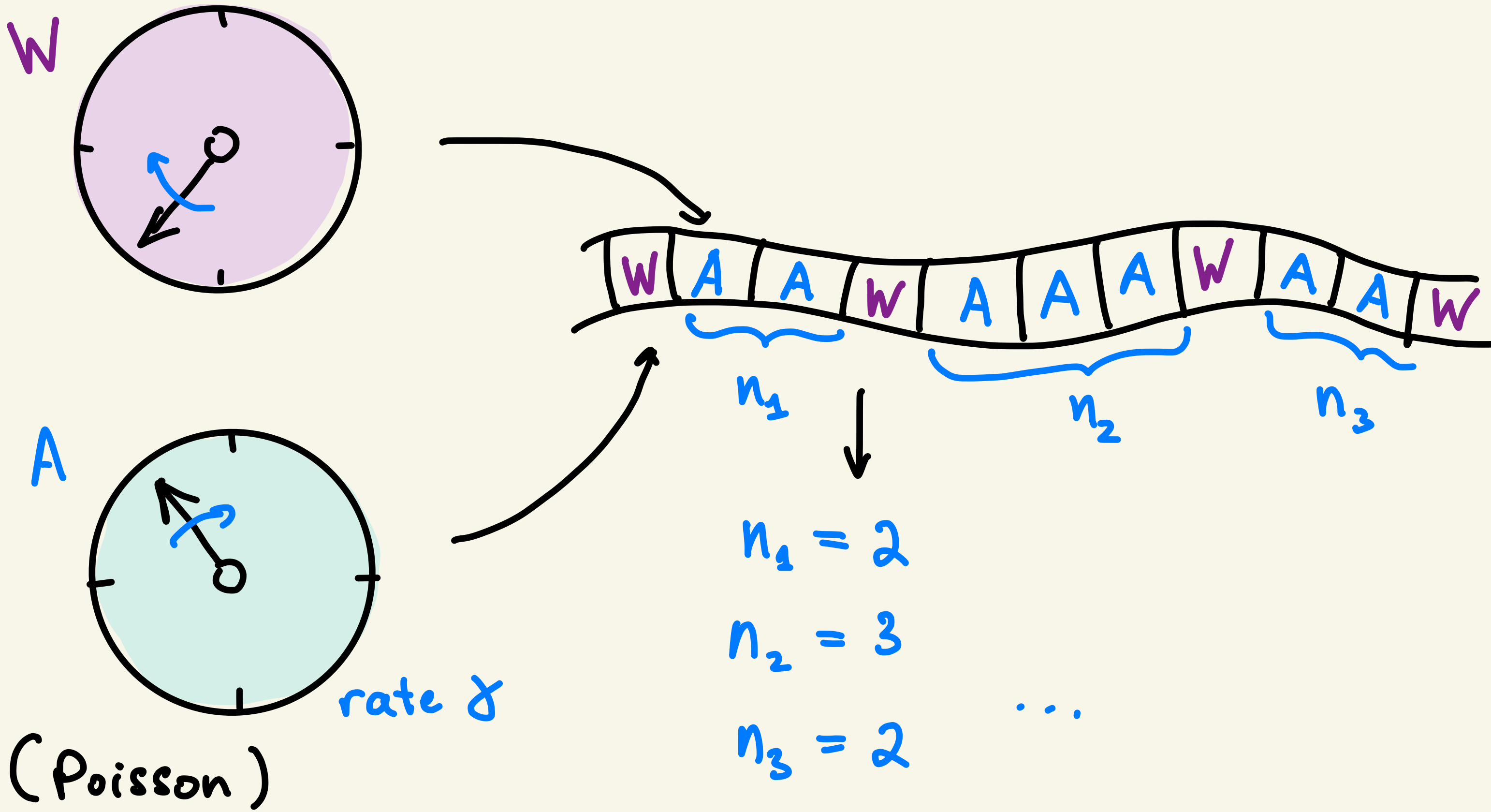


(Poisson) (arbitrary iid clock)

$$P(T_1 = t_1, T_2 = t_2, \dots) = w(t_1) \cdot w(t_2 - t_1) \cdot \dots$$

$$w(t) = ?$$

Arbitrary clock vs. Poisson clock



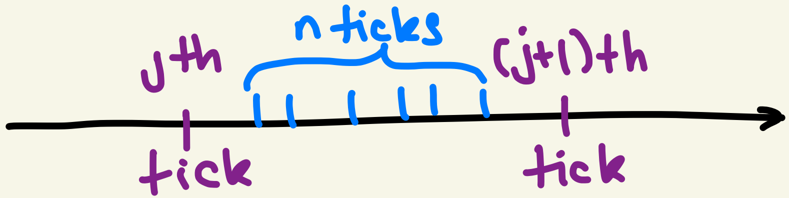
$n_1 = 2$

$n_2 = 3$

$n_3 = 2$

...

Moments relative to Poisson clock



$$P(N_A = n, T_{j+1} - T_j = t) = P(N_A = n, t) \cdot w(t)$$

$$\downarrow$$

$$p_n = \int_0^{+\infty} \underbrace{P(N_A = n, t)}_{\frac{(\gamma t)^n e^{-\gamma t}}{n!}} \, w(t) \, dt$$

Moments relative to Poisson clock

$$m_k(\omega, \gamma) = \sum_{n=0}^{\infty} n^k p_n$$

Stirling number of 2^{nd} kind

$$= \sum_{j=1}^{k} \left\{ {k \atop j} \right\} \gamma^j M_j(\omega)$$

for example:

$$m_1(\omega, \gamma) = \gamma M_1(\omega)$$

$$m_2(\omega, \gamma) = \gamma^2 M_2(\omega) + \gamma M_1(\omega)$$

$$\cdots$$

Moments relative to Poisson clock

$$m_k(\omega, \gamma) = \sum_{n=0}^{\infty} n^k p_n$$

Stirling number of 2^{nd} kind

$$= \sum_{j=1}^{k} \left\{ {k \atop j} \right\} \gamma^j M_j(\omega)$$

$$M_k(\omega) = \frac{1}{\gamma^k} \sum_{j=1}^{k} (-1)^{k-j} \left[{k \atop j} \right] m_j(\omega, \gamma)$$

Stirling numbers of 1^{st} kind

Moments relative to Poisson clock

Characteristic function

$$\langle e^{ixT} \rangle_{\omega(t)} = \sum_{k=0}^{\infty} \frac{(ix)^k}{k!} M_k(\omega)$$

uniquely defines $\omega(t)$!

Finite experimental samples

Sample size N with $X_1, X_2, \dots, X_N$

(for calculating first relative moment)

$$P\left(\left| \frac{\bar{X}}{\gamma} - \frac{m_1(\omega,\gamma)}{\gamma} \right| \geq \epsilon \right) \leq \frac{m_2(\omega,\gamma) - m_1^2(\omega,\gamma)}{N \gamma^2 \epsilon^2}$$

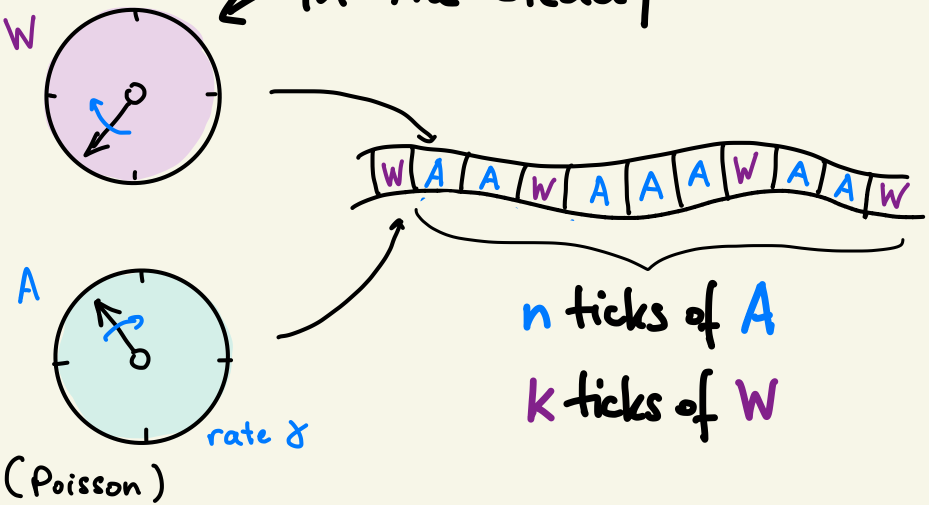
σ_w^2

$$= \frac{M_2(\omega) - M_1^2(\omega)}{N \epsilon^2} + \frac{1}{N\epsilon^2} O\left(\frac{1}{\gamma}\right)$$

γ is a sampling resource!

Tomography of non-reset clocks

in the steady state



$$P(k|n) = \int_0^{+\infty} P_A(n|t)\, P_W(k|t)\, dt$$

relative moments

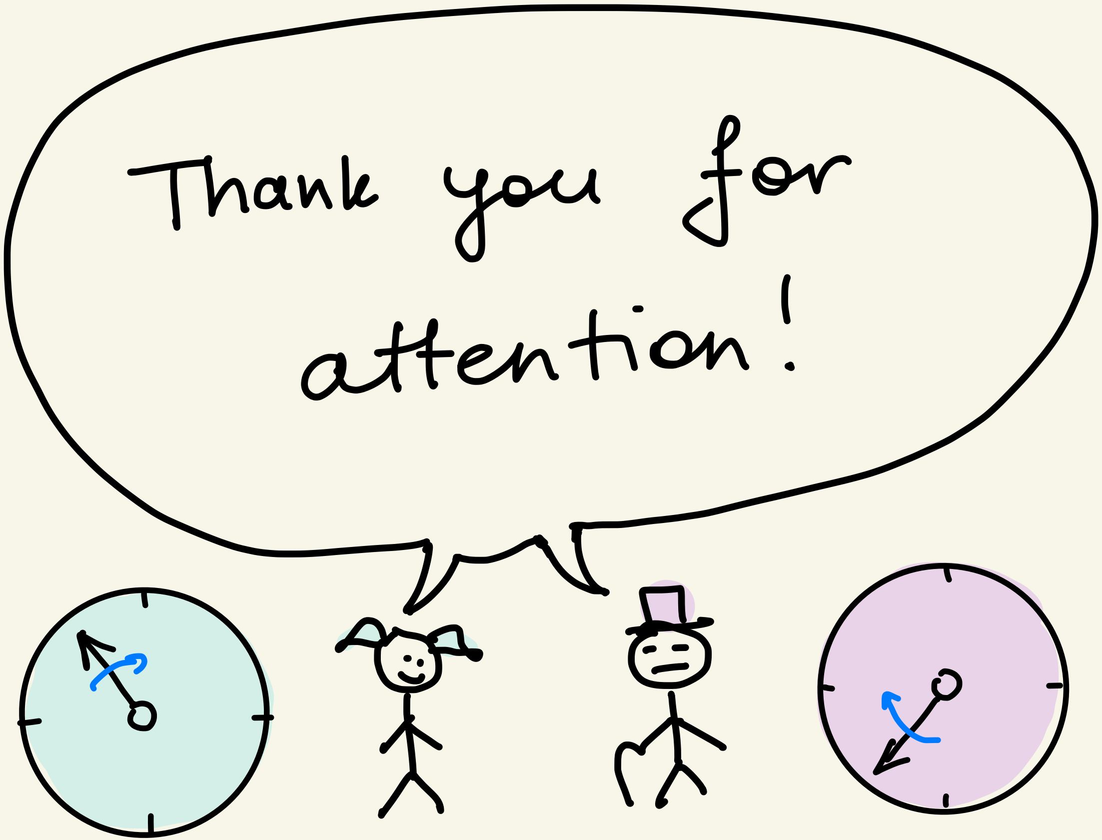
$$m_j(\gamma, n) = \sum_{k=0}^{\infty} k^j P(k|n)$$

asymptotics $\langle X^j \rangle = \sum_{k=0}^{\infty} k^j P(k|t) = \sum_{s=0}^{j} \alpha_s^{(j)} t^s$

$\rightarrow$ find moments from $\{\alpha_s^{(j)}\}_{s,j}$

Outlook

* general clock tomography:
  - arbitrary vs. arbitrary
    known unknown
  - arbitrary vs. arbitrary (e.g. same clocks)
    unknown unknown
  - relativistic settings


Thank you for attention!

Assumptions for deriving the simplest clock model