

Macrorealism as strict classicality in the framework of generalized probabilistic theories

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Quantum Mechanics versus Macroscopic Realism: Is the Flux There when Nobody Looks?

A. J. Leggett

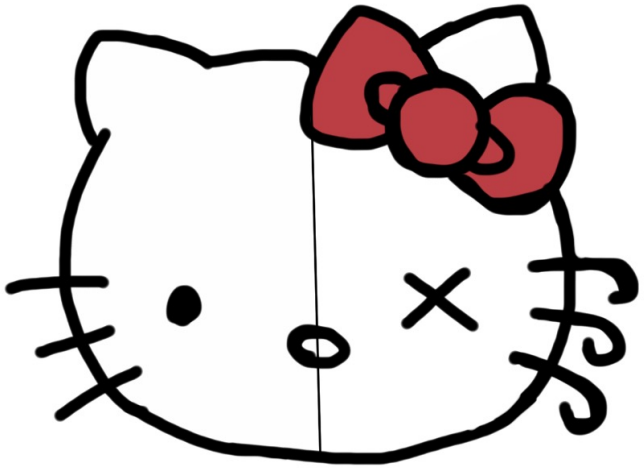
*Department of Physics,^(a) University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, and Department of Physics,
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and

Anupam Garg

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(Received 19 November 1984)



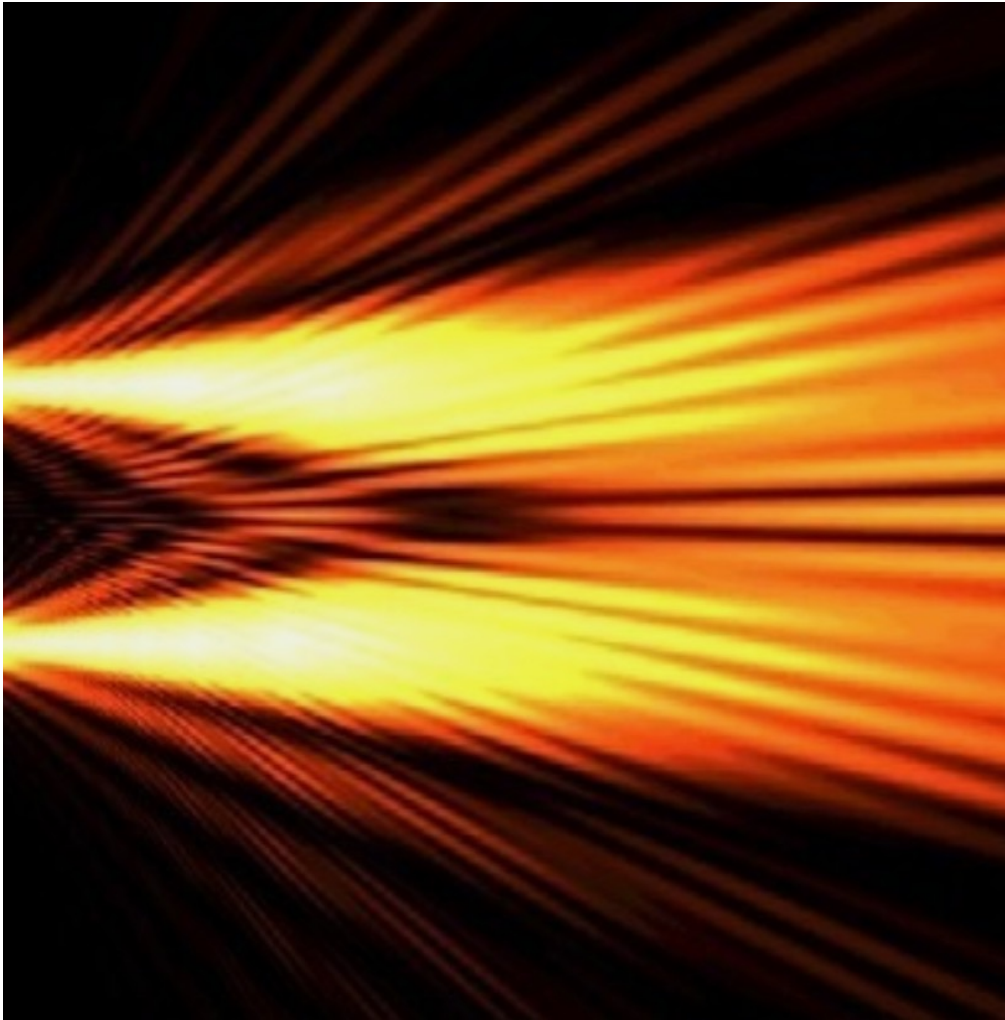
“Despite sixty years of schooling in quantum mechanics, most physicists have a very non-quantum-mechanical notion of reality at the macroscopic level.”

-LG

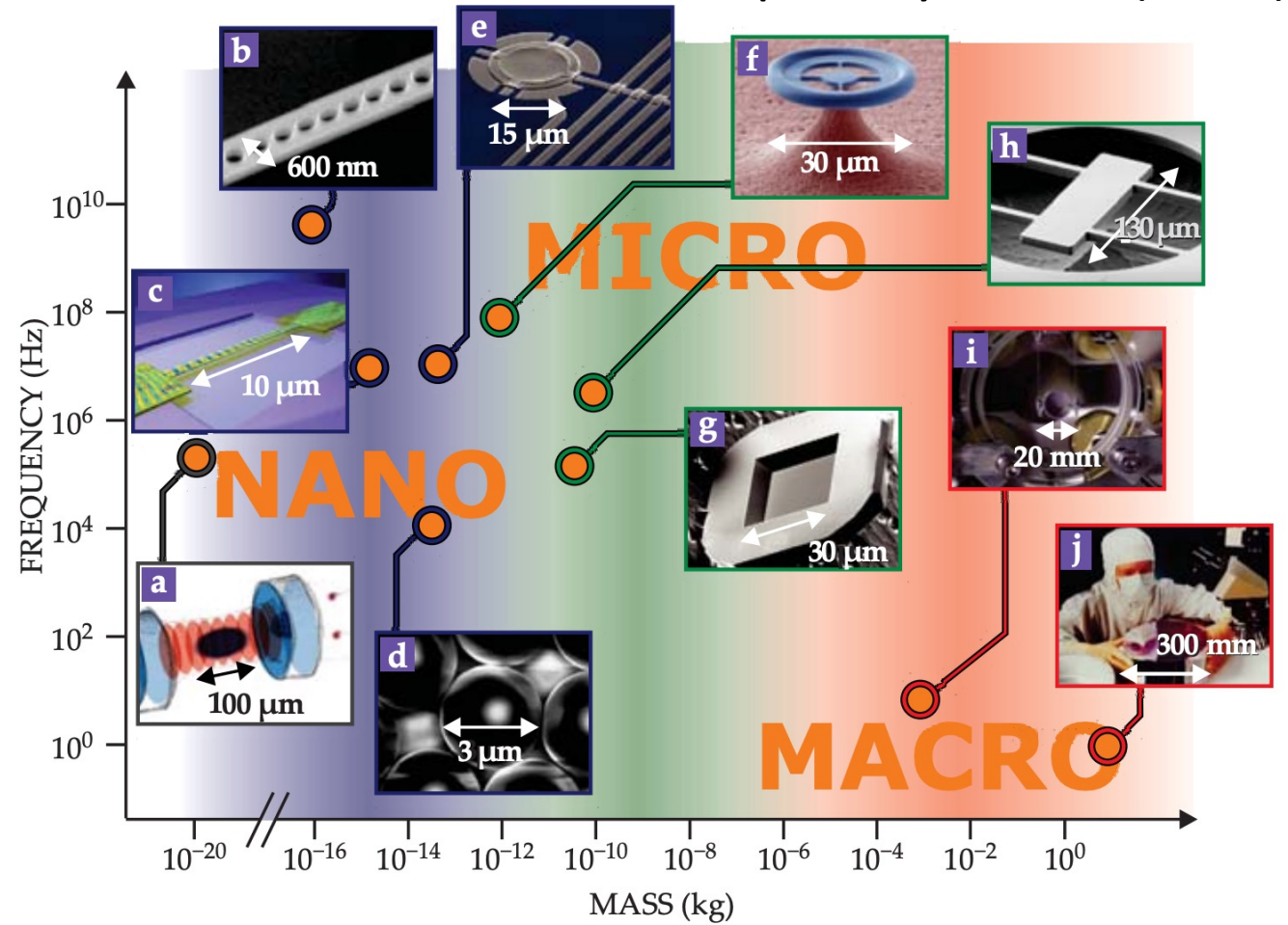
Objects in the “everyday world of our immediate experience” have definite states and can be observed without disturbance.

So... does quantum theory break down in the macroscopic limit?





Aspelmeyer et al (2012)



LG were not satisfied: where is the *formal proof* that interference can't be explained in a classical theory?

(indeed, see arXiv:2111.13727)

1. Give a rigorous definition of classicality
 - a. Leggett-Garg's definition of macrorealism
 - b. Better definition of macrorealism

2. Devise an experiment that is capable of ruling out this notion of classicality.
 - a. Leggett-Garg's (and others) experimental proposals
 - b. Better experimental proposal

LG's definition

Macrorealism per se. A macroscopic object which has available to it two or more macroscopically distinct states is at any given time in a definite one of those states.

Noninvasive measurability (at the macro level). It is possible in principle to determine which of these states the system is in without any effect on the state itself or on the subsequent system dynamics.



Macrorealism

LG's definition

Macrorealism per se. A **macroscopic** object which has available to it two or more **macroscopically** distinct states is at any given time in a definite one of those states.

Noninvasive measurability (at the **macro level).** It is possible in principle to determine which of these states the system is in without any effect on the state itself or on the subsequent system dynamics.



Macrorealism

Note that this definition works for any notion of macroscopicity.

Clarifying the definition of macrorealism

A nice analysis, which my arguments build on:

Quantum- vs. Macro- Realism: What does the
Leggett-Garg Inequality actually test?

O.J.E Maroney*

Faculty of Philosophy and Wolfson College, Oxford

C.G Timpson†

Faculty of Philosophy and Brasenose College, Oxford

December 22, 2014

Macrorealism per se. A macroscopic object which has available to it two or more macroscopically distinct **states** is at any given time in a definite one of those states.

What is the notion of “state” being used here?

Quantum state?

Ontic state?

Operational state?

Leggett and Garg's discussion strongly suggests that their background framework is simply quantum mechanical Hilbert space states. *Macroscopic states are the quantum states that one would assign to macroscopic, or collective, degrees of freedom.* Thus, in a SQUID, one does not trouble to assign a (massively entangled) multi-particle quantum state to the *enormous* number of individual microscopic charge-carriers, rather one simply assigns a single state to the collective degree of freedom, the direction of the current, e.g., $|+1\rangle$ or $|-1\rangle$. The content of macroscopic realism is then that the only permissible states of the SQUID are the quantum states $|+1\rangle$ and $|-1\rangle$ (and statistical mixtures thereof), quantum superpositions of these two states being disallowed.

--Timpson and Maroney

The notion of state is basically the quantum state, but not all quantum states are allowed—only the diagonal ones.

$$\begin{bmatrix} \alpha & 0 \\ 0 & 1 - \alpha \end{bmatrix}$$

Leggett:

...in this [quantum] language, the predictions of a macrorealist theory... are equivalent to those which follow from putting C [the off-diagonal terms in the density matrix] to zero. (Leggett et al, 2016, p. 4)

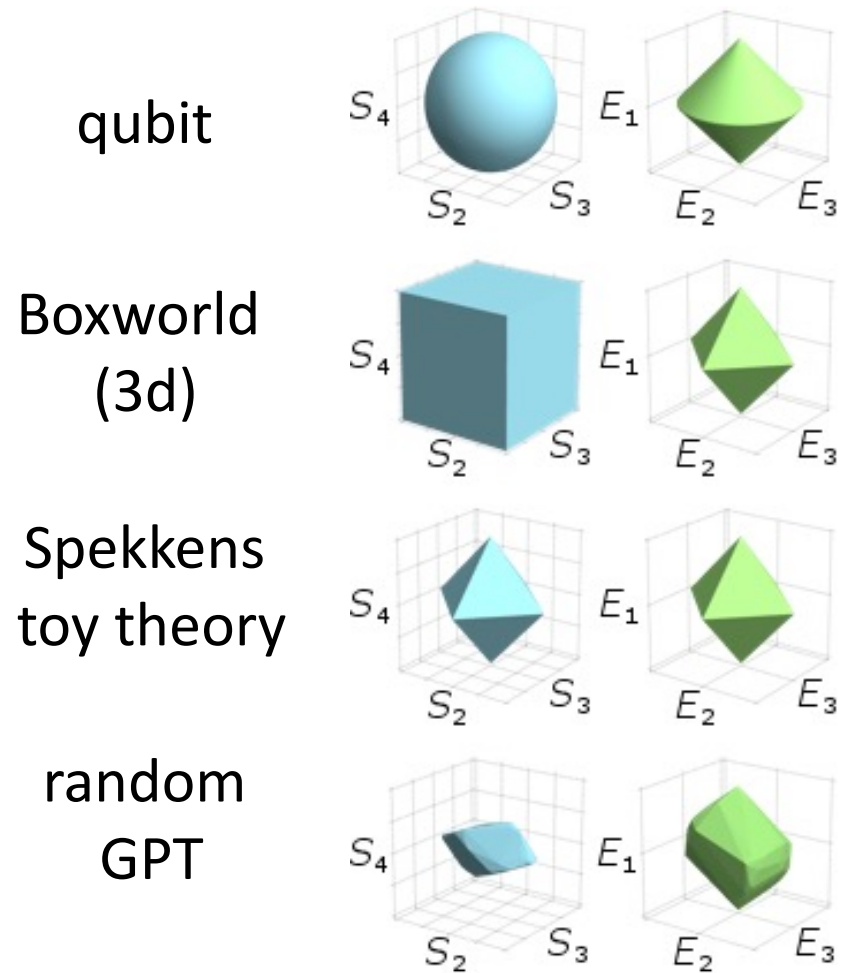
macrorealism \cong diagonal quantum theory

A very simple idea... but hard to state without reference to quantum concepts!
...or, it *was*, before we had the framework of generalized probabilistic theories (GPTs)!

An update to this old, vague idea is way overdue!

Generalized Probabilistic Theories

Generalized Probabilistic Theories (GPTs)

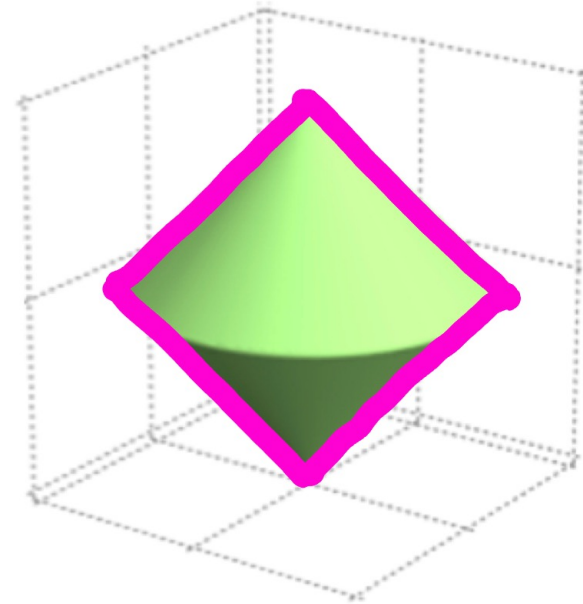
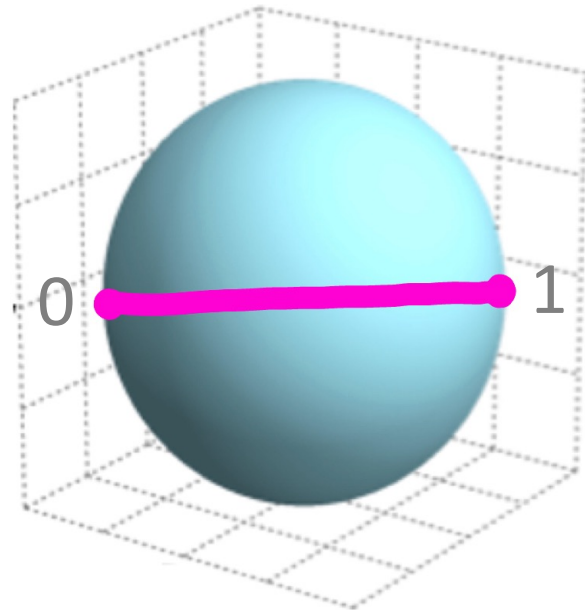


content of a theory = convex geometry
states and effects = real valued vectors
empirical probabilities = dot products

“Diagonal quantum theory” as a GPT?

simplest case: a totally dephased qubit

d=2
simplicial GPT



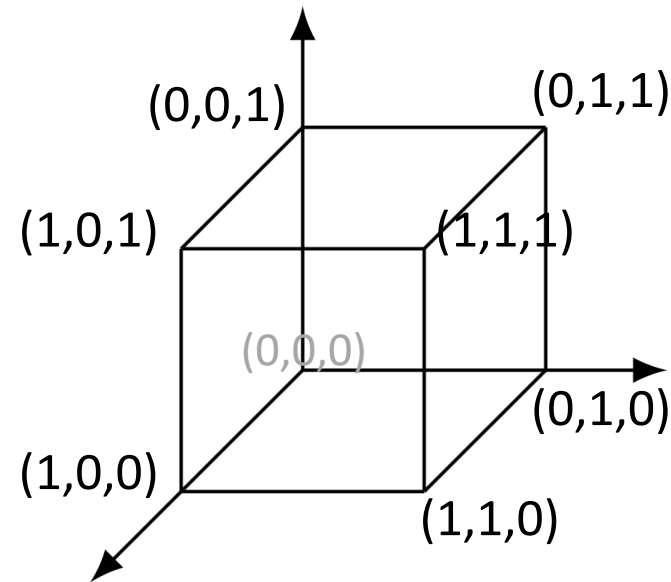
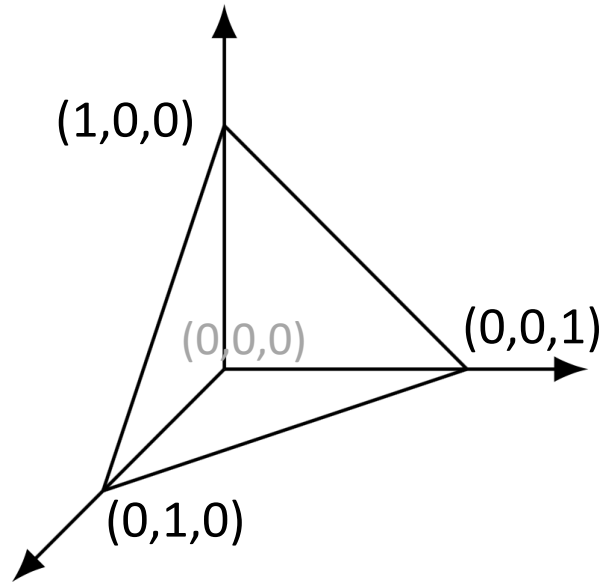
classical bit: two pure states that are perfectly distinguishable, and all mixtures thereof

“Diagonal quantum theory” \mapsto Simplicial GPT

state space: simplex

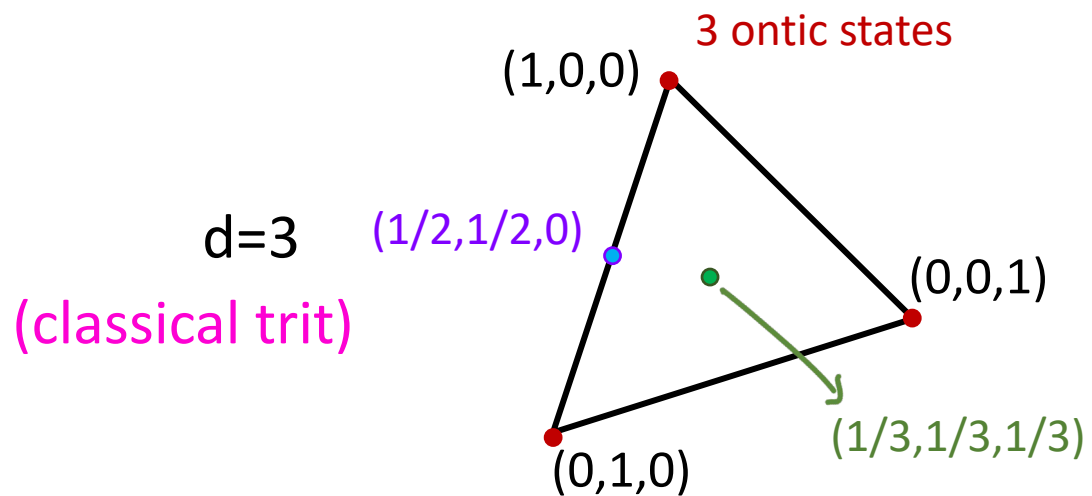
effect space: dual of simplex

d=3

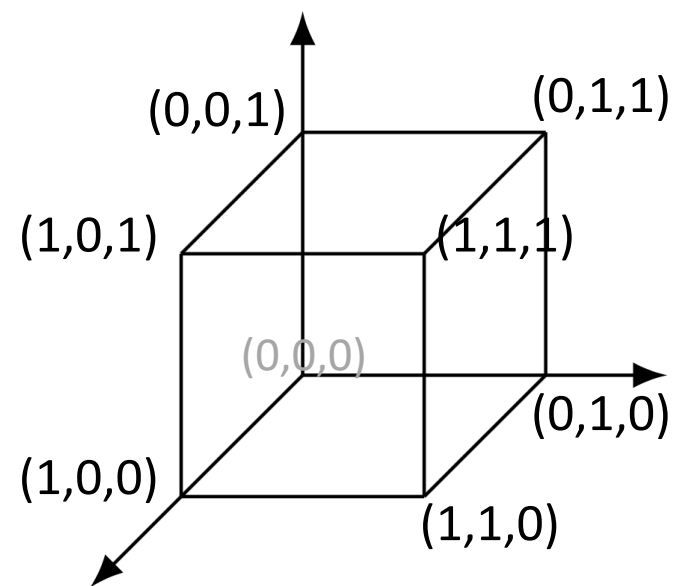


simplicial = strictly classical
(classical statistical theory)

normalized states

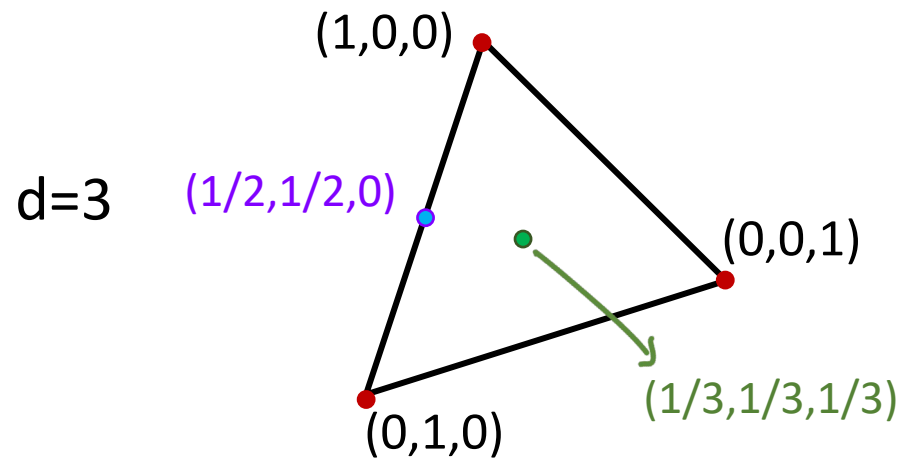


effects

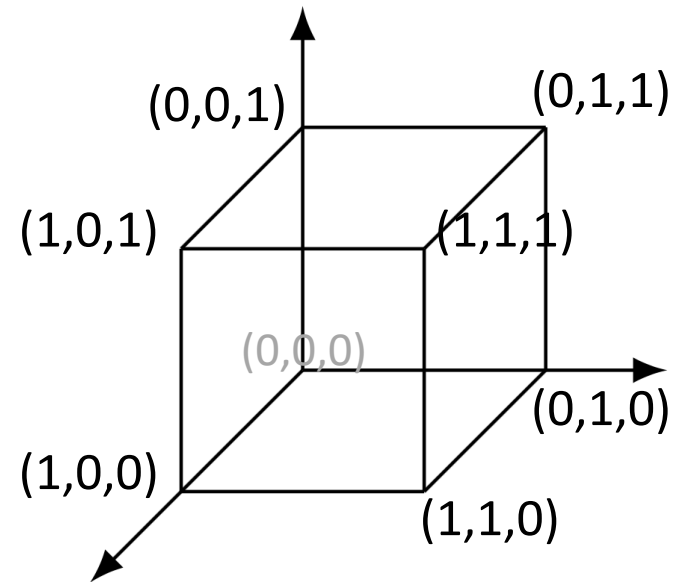


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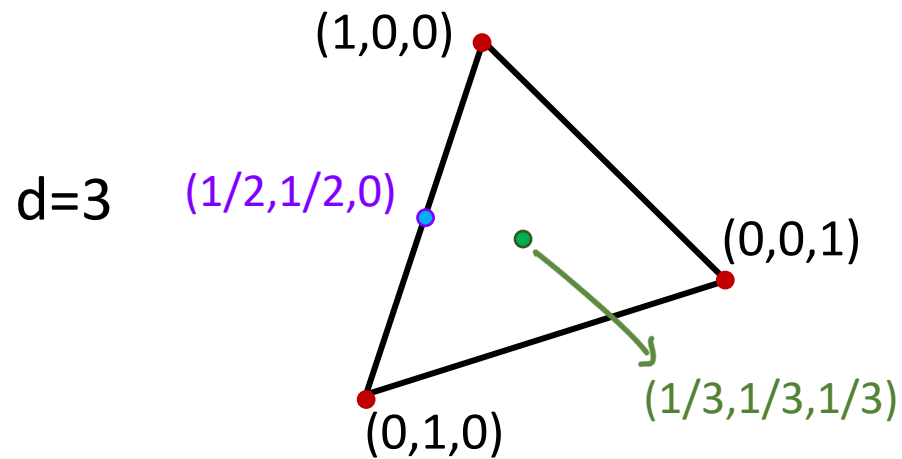


effects

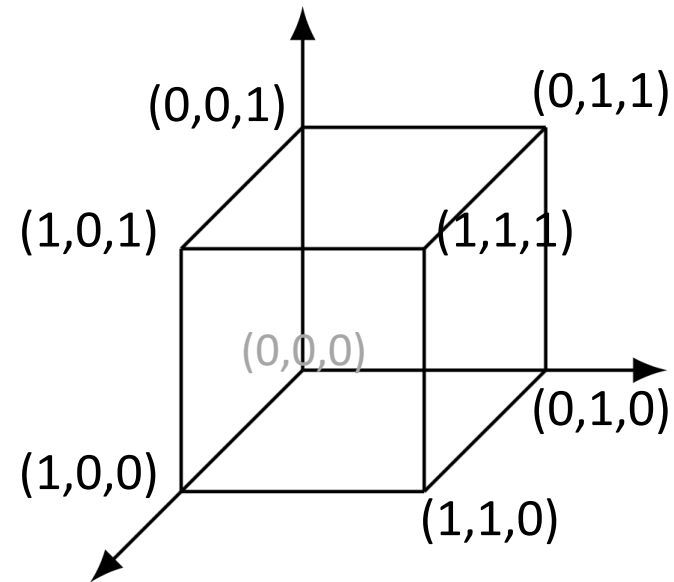


Every mixed state decomposes into pure states in a unique way
 \Rightarrow One can always imagine that there is a true state of the system, and any mixed state can be *uniquely* interpreted as uncertainty about the true state.

normalized states



effects



All logically possible measurements are *physically possible* and *compatible*.
⇒ One can determine the exact state of the system in a single measurement.

More properly, a GPT should be defined with transformations, measurements, etc.

Transformations in the simplicial GPT correspond to stochastic maps on the vertices.

The simplicial GPT contains a nondisturbing and perfectly informative measurement.

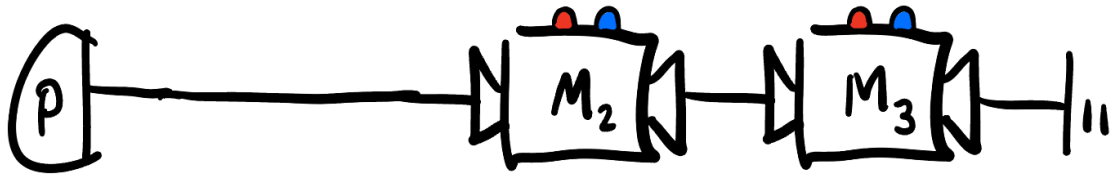
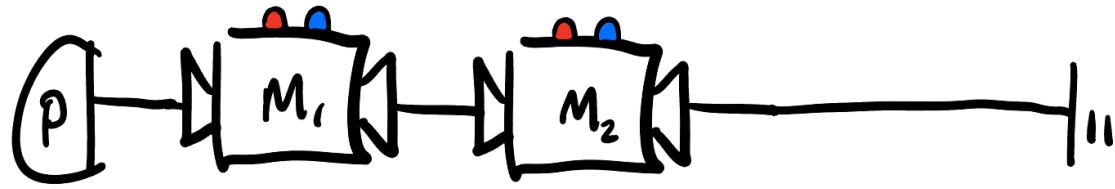
This formalizes Noninvasive Measurability!

Macrorealism is best characterized as the operational hypothesis that macroscopic systems are described by strictly classical GPTs.

Key advantages of this definition over LG's

1. No ambiguity about the notion of state
2. Makes explicit the fact that macrorealism is an operational hypothesis
3. Makes explicit the fact that macrorealism is a notion of classicality, not of realism
4. Is a full-fledged characterization of a theory
5. Allows us to directly apply tools from framework of GPTs
6. Allows us to leverage known facts about simplicial GPTs
7. Allows us to relate macrorealism and generalized noncontextuality

Tests of Macrorealism



Assuming M_1 and M_2 are nondisturbing:

$$-1 \leq \langle Q_1 Q_2 \rangle_{M_1 M_2} + \langle Q_1 Q_3 \rangle_{M_1 M_3} + \langle Q_2 Q_3 \rangle_{M_2 M_3}$$

Leggett-Garg inequality

Much more general tests of LG's assumptions are possible.

(Clifton)

Any interference experiment can be used to test LG's assumptions.

“no-signaling in time equalities”

(Kofler, Brukner)

Many physicists have challenged the noninvasiveness assumption:

O. J. Maroney and C. G. Timpson, arXiv:1412.6139 (2014).

L. Hardy, D. Home, E. J. Squires, and M. A. B. Whitaker, Phys. Rev. A 45, 4267 (1992).

S. Foster and A. Elby, Foundations of Physics 21, 773 (1991).

F. Benatti, G. Ghirardi, and R. Grassi, Foundations of Physics Letters 7, 105 (1994).

R. Clifton, Symposium on the foundations of modern physics (World Scientific, 1990).

G. Bacciagaluppi, arXiv:1409.4104 (2014).

etc

Why interference phenomena do not capture the essence of quantum theory

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Leggett and Garg try to
motivate noninvasiveness via
null-result measurements

Not convincingly!

theory-dependent assumptions

⇒

not methodologically on par with Bell's theorem

Knee, Leggett, et al. (2016): instead of assuming noninvasiveness, one can quantify the invasiveness of the measurements using a control experiment

...but now they must assume perfectly known and perfectly pure control **states**

The GPT framework easily allows one to construct counterexamples to their "proof".

Of course, one could try to *quantify* the purity of the preparations...
But only if we assume we have well-characterized measurements to do the quantification!

Catch-22?

Better tests of macrorealism:
theory-agnostic GPT tomography

How does one determine the GPT describing a given system
...without assuming one already has access to characterized states or mmts?

PRX QUANTUM 2, 020302 (2021)

**Experimentally Bounding Deviations From Quantum Theory in the Landscape of
Generalized Probabilistic Theories**

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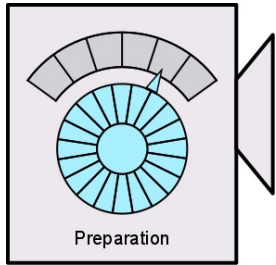
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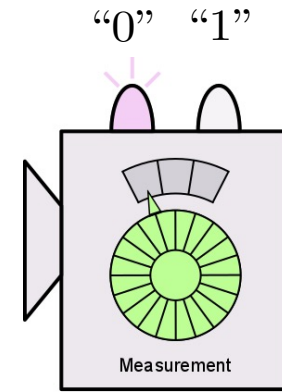
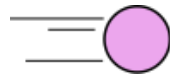
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P_1, \dots, P_m



M_1, \dots, M_n

$$\left(\begin{array}{cccccc} 1 & p(0|P_1, M_2) & p(0|P_1, M_3) & p(0|P_1, M_4) & p(0|P_1, M_5) & \dots \\ 1 & p(0|P_2, M_2) & p(0|P_2, M_3) & p(0|P_2, M_4) & p(0|P_2, M_5) & \dots \\ 1 & p(0|P_3, M_2) & p(0|P_3, M_3) & p(0|P_3, M_4) & p(0|P_3, M_5) & \dots \\ 1 & p(0|P_4, M_2) & p(0|P_4, M_3) & p(0|P_4, M_4) & p(0|P_4, M_5) & \dots \\ 1 & p(0|P_5, M_2) & p(0|P_5, M_3) & p(0|P_5, M_4) & p(0|P_5, M_5) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array} \right)$$

m by K

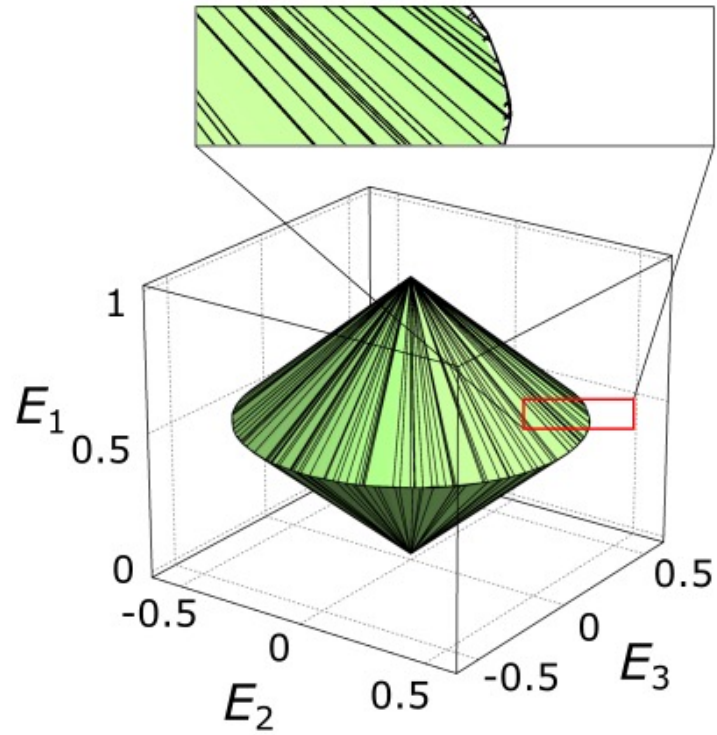
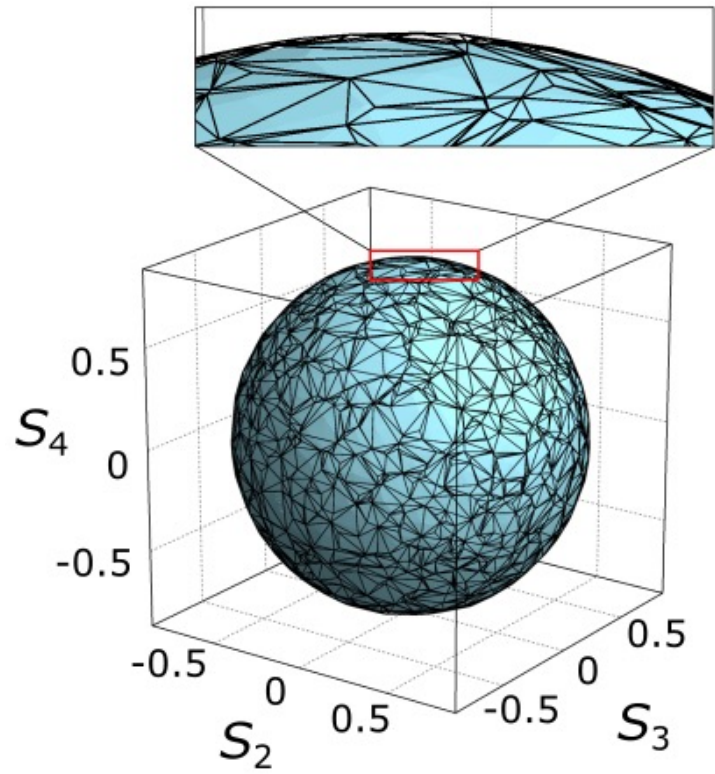
K by n

GPT state

$$\begin{pmatrix} 1 & P_1^{(2)} & \dots & P_1^{(K)} \\ 1 & P_2^{(2)} & \dots & P_2^{(K)} \\ 1 & P_3^{(2)} & \dots & P_3^{(K)} \\ 1 & P_4^{(2)} & \dots & P_4^{(K)} \\ 1 & P_5^{(2)} & \dots & P_5^{(K)} \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} 1 & M_{2,0}^{(1)} & M_{3,0}^{(1)} & M_{4,0}^{(1)} & M_{5,0}^{(1)} & \dots \\ 0 & M_{2,0}^{(2)} & M_{3,0}^{(2)} & M_{4,0}^{(2)} & M_{5,0}^{(2)} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots \\ 0 & M_{2,0}^{(K)} & M_{3,0}^{(K)} & M_{4,0}^{(K)} & M_{5,0}^{(K)} & \dots \end{pmatrix}$$

GPT effect

$$p(0|P_i, M_j) = \mathbf{s}_i \cdot \mathbf{e}_{j,0} = \begin{pmatrix} 1 & P_i^{(2)} & \dots & P_i^{(K)} \end{pmatrix} \cdot \begin{pmatrix} M_{j,0}^{(1)} & \dots & M_{j,0}^{(K)} \end{pmatrix}$$



Does the GPT fit inside a simplex and its dual?

If not, then macrorealism is falsified!

Key advantages over prior proposals:

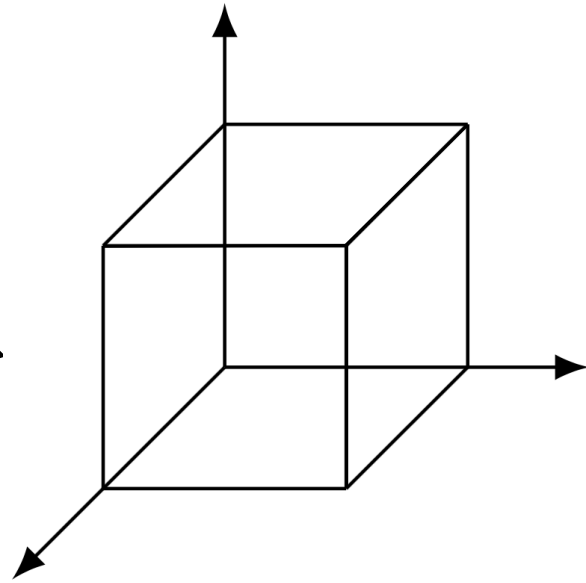
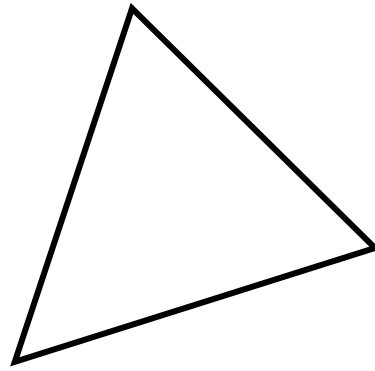
1. Don't need to make noninvasive measurements
2. Don't need to prepare pure states
3. Don't need prior characterization of *either* preparations or measurements
4. One can gain a full characterization of the DOF, not just a single witness
5. One can use the same data to test for noncontextuality as well

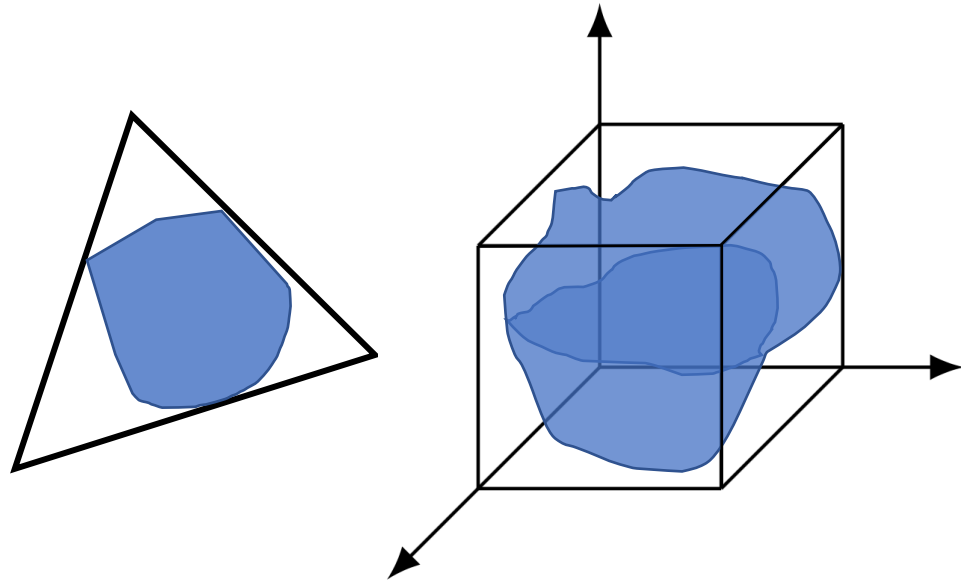
Macrorealism versus Noncontextuality

Given a theory or a set of data, generalized noncontextuality provides a principled way to decide if there is any classical explanation or not.

Noncontextual \cong GPT is Simplex-embeddable

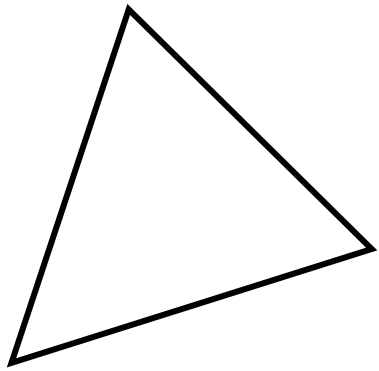
(prepare-measure scenarios)



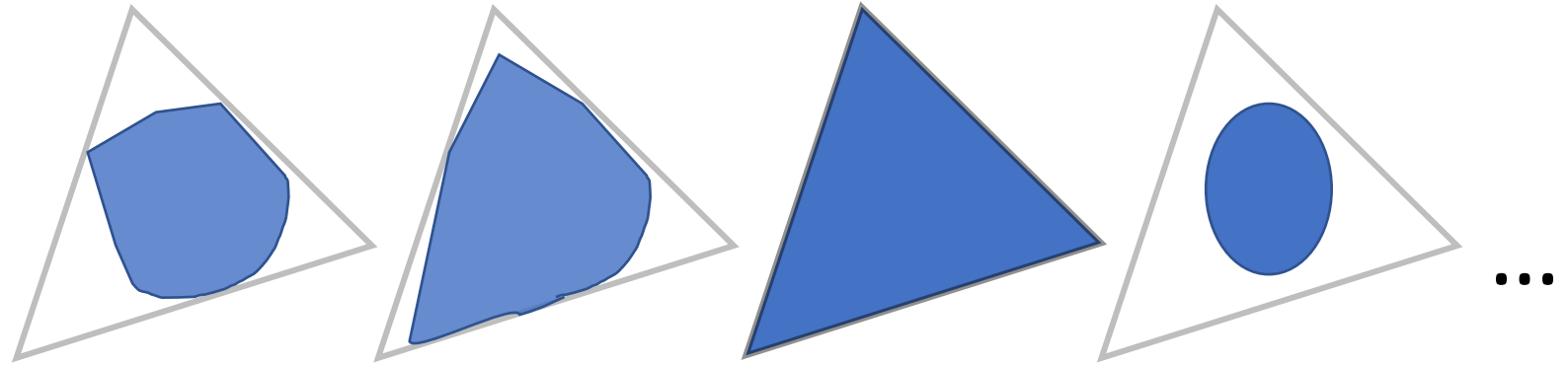


“subsystems” of a simplicial GPT

Macrorealist \Rightarrow Noncontextual



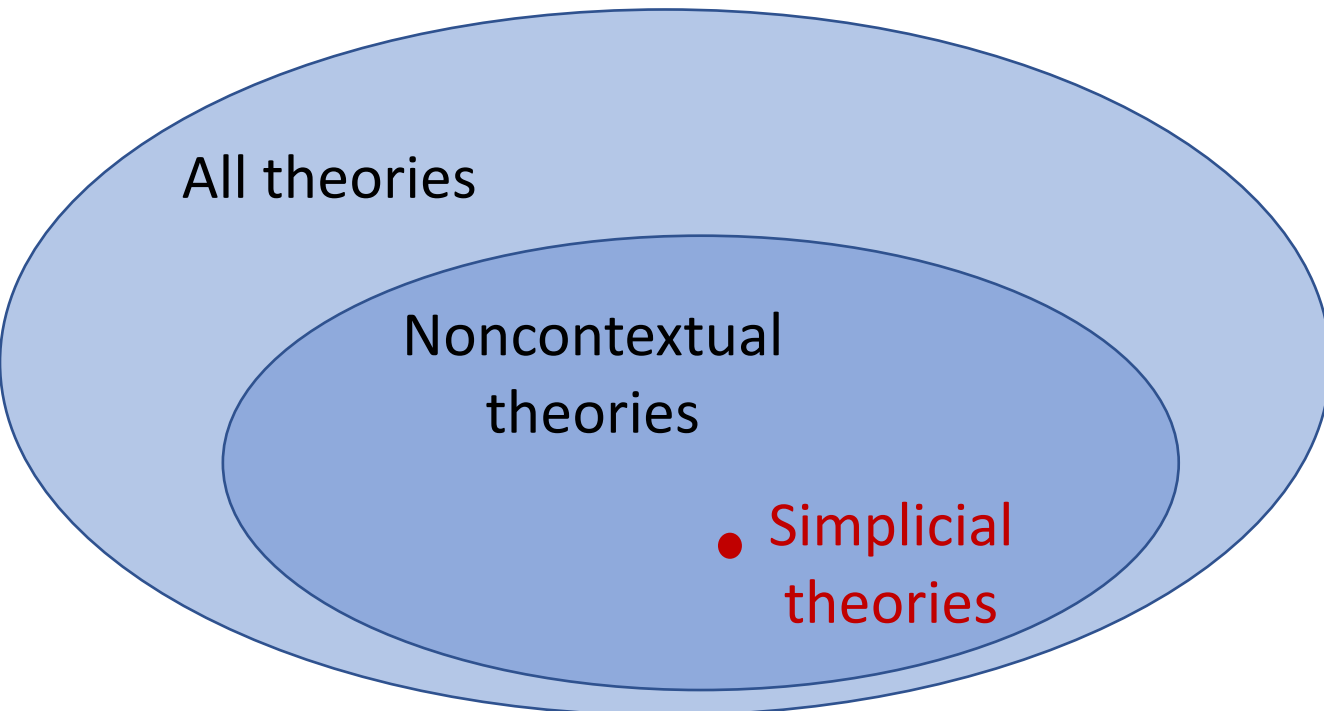
Macrorealist



Noncontextual

A violation of **MR** establishes a **weak** form of nonclassicality.
A violation of **NC** establishes a **strong** form of nonclassicality

Theory-agnostic tomography can be used for both kinds of tests!



So, any violation of generalized noncontextuality on a macroscopic system implies the failure of macrorealism

Macrorealism \mapsto Strict simpliciality (for macroscopic systems)

Leggett-Garg tests \mapsto Theory-agnostic tomography

arXiv:2209.11783

**Macrorealism as strict classicality in the framework of
generalized probabilistic theories (and how to falsify it)**

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