Causal inference in non-classical theories and compatibility with space-time structure

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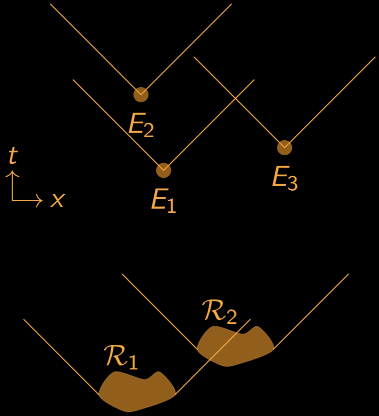
Joint work with Roger Colbeck (University of York, U.K.)

- V. Vilasini and Roger Colbeck. PRA, 106, 042204 (2022).
- V. Vilasini and Roger Colbeck. PRL, 129, 110401 (2022).

Different notions of causality

Spatio-temporal notions



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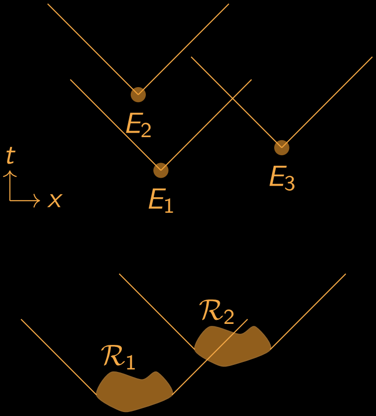
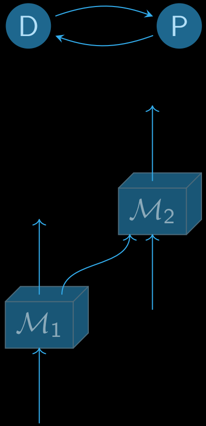


Information-theoretic notions



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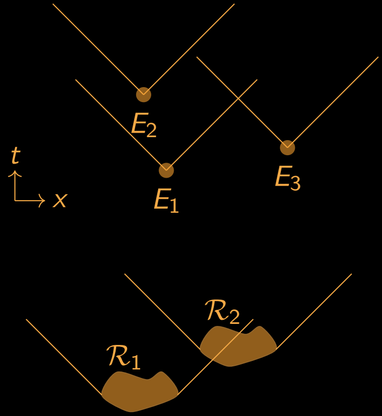
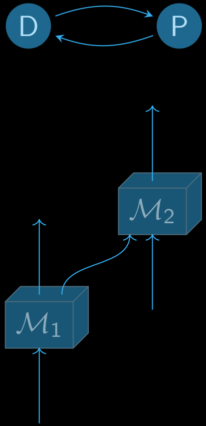
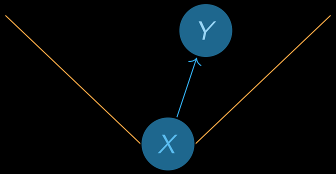




In physical experiments, these notions must play together!

Relativistic causality principles

E.g., No signalling outside the future lightcone



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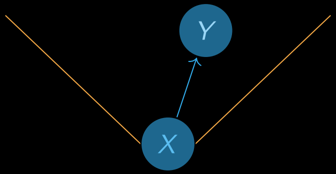

These are about the compatibility between spatiotemporal and information-theoretic causal order relations

How can we formulate compatibility between information-theoretic and spatio-temporal causal structures?

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Applicable to two classes of information-theoretic causal structures

1. Causal models (possibly non-classical, cyclic) (today)
2. Indefinite causal structures (QPL 2022)

Structure of this talk:

1. Framework: Causal modelling and causal inference

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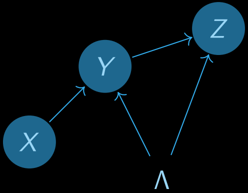
4. Conclusion and outlook

Causal models and causal inference: motivation

Observable data $\xrightarrow{\text{Causal inference}}$ Cause-effect relations

Observable data $\xleftarrow[\text{Causal modelling}]{}$ Cause-effect relations

E.g., $P(XYZ)$



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Observable data

Causal inference $\longrightarrow$

$\longleftarrow$ Causal modelling

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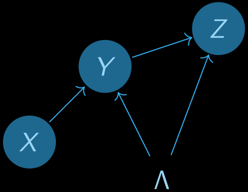

Quantum correlations challenge classical causal explanations

$\Rightarrow$ Develop quantum (/non-classical) causal models

Pearl, 2000 and 2009. Spirtes, 2001. Wood and Spekkens 2015.

Causal structures and correlations

- Causal structure: Directed graph $\mathcal{G}$.

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Henson, Lal, Pusey. 2014. Vilasini and Colbeck 2022.

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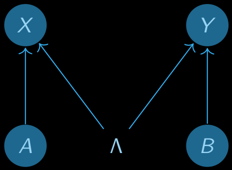
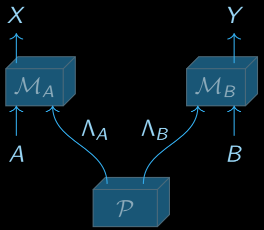
Causal structure: $\mathcal{G}^{Bell}$



Causal mechanisms



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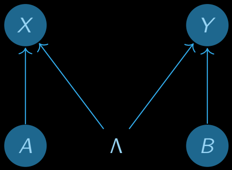

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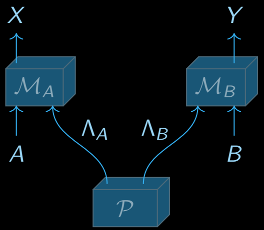

Observed distribution: Joint probability distribution $P_{\mathcal{G}}(S_{obs})$ over all observed nodes S_{obs} of $\mathcal{G}$. For $\mathcal{G}^{Bell}$, $P(XYAB)$.

Henson, Lal, Pusey. 2014. Vilasini and Colbeck 2022.

Constraints on $P_{\mathcal{G}}(S_{obs})$ from $\mathcal{G}$

- Theory-dependent:
- Theory-independent:

Henson, Lal, Pusey 2014. Bell 1964. Pearl 2009.

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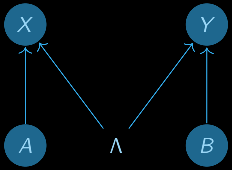
E.g., Non-signalling constraints in $\mathcal{G}^{Bell}$



The d-separations $X \perp^d B|A$ and $Y \perp^d A|B$ imply $P(X|AB) = P(X|A)$ and $P(Y|AB) = P(Y|B)$.

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Several different quantum causal modelling/inference frameworks

- Bottom-up*:

- Top-down**:

*Liefer Spekkens 2013, Hensen, Lal, Pusey 2014, Pienaar 2015, Costa and Shrapnel 2016, Barrett, Lorenz, Oreshkov 2020 and 2022.

**Vilasini and Colbeck 2022

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**Vilasini and Colbeck 2022

Several different quantum causal modelling/inference frameworks

- Bottom-up*: Start with assumptions on causal mechanisms and derive *d*-separation and other properties (often acyclic)
- Top-down**: Start by assuming *d*-separation on S_{obs} and derive consequences for causal mechanisms (also cyclic)

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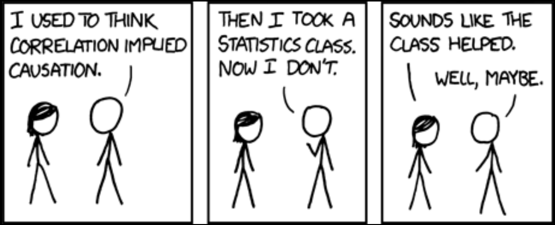
**Vilasini and Colbeck 2022

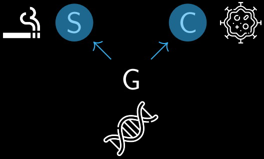
Interventions and affects relations



Correlation alone can't single out a causal explanation, need interventions!

(a)

(b)



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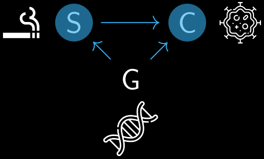
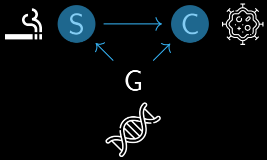
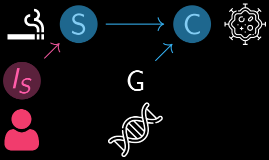


(d)

Pre-intervention: $\mathcal{G}$



Post-intervention: $\mathcal{G}_{\mathrm{do}(S)}$



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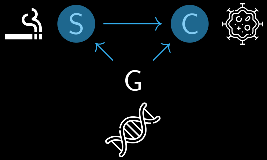

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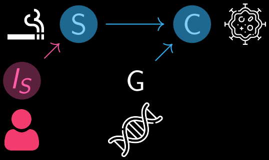

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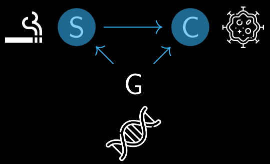

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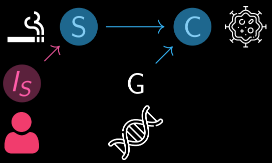

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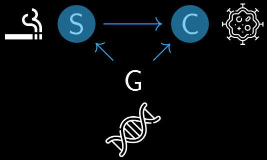

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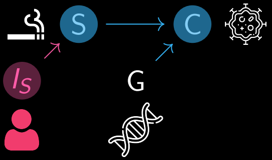

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In general, $P_{\mathcal{G}_{\mathrm{do}(S)}}(C|S) \neq P_{\mathcal{G}}(C|S)$ except if S is parentless

In paper: rules for relating $P_{\mathcal{G}_{do(S)}}$ and $P_{\mathcal{G}}$ in cyclic, non-classical causal models

Causal inference: X affects $Y \Rightarrow X$ is a cause of Y in $\mathcal{G}$

But converse is NOT true! E.g., one-time pad

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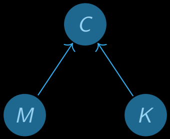
M, K, C binary RVs



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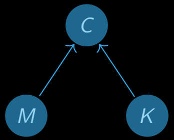

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$\Rightarrow$ M is a cause of C but M does not affect C

Higher-order (HO) affects relation: X affects Y given do(Z)

Captures signalling between sets of RVs X and Y when given an intervention performed on another set Z of RVs

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In paper: Causal inference results for HO affects, can infer more.

Embedding causal models in space-time

Space-time: partially ordered set $\mathcal{T}$ with order relation $\preceq$

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Space-time embedding $\mathcal{E}$: $X \in S_{obs} \mapsto O(X) \in \mathcal{T}$.
Leads to ordered random variable (ORV), $\mathcal{X} = (X, O(X))$.

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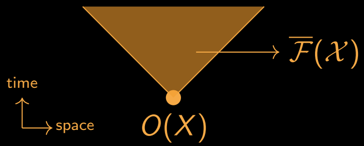
Inclusive future of an ORV: $\overline{\mathcal{F}}(\mathcal{X}) := \{P \in \mathcal{T} | P \succeq O(X)\}$



Compatibility: Ensures no signalling outside space-time future
(In particular: X affects $Y \Rightarrow \overline{\mathcal{F}}(Y) \subseteq \overline{\mathcal{F}}(X)$ for RVs)

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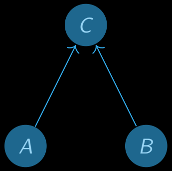

$C = A \oplus B$, B uniform

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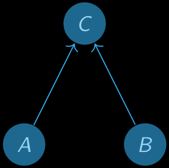

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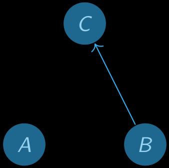

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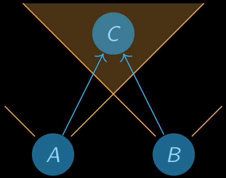

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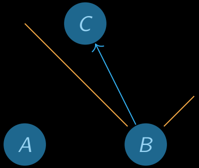

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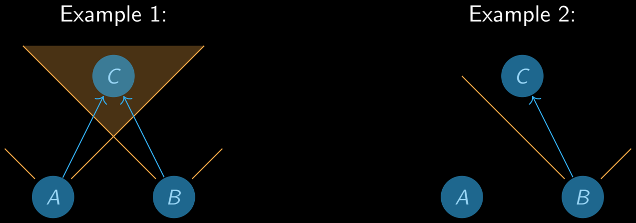

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Can't infer compatibility conditions from interventional data?

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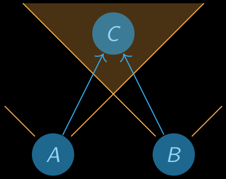

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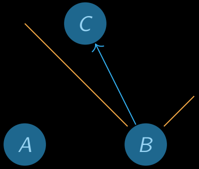

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Can't infer compatibility conditions from interventional data?

Solution: We can, using higher-order affects relations

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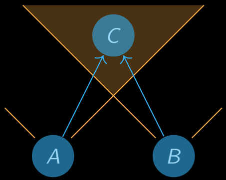

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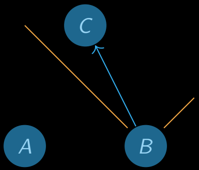

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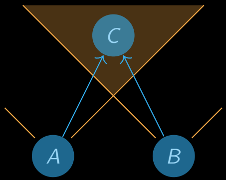

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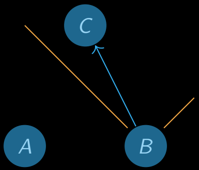

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- Affects rel.: B affects C, AB affects C (A <u>does not affect</u> C)
- Higher-order affects rel: A affects C given $\mathrm{do}(B)$ in 1, not 2

$\Rightarrow$ Infer A is a cause of C in Ex 1 but not in Ex 2
$\Rightarrow \overline{\mathcal{F}}(\mathcal{C}) \subseteq \overline{\mathcal{F}}(\mathcal{A})$ for compat in Ex 1 but not Ex 2

Example 3: Jamming non-local correlations*
Signalling jointly but not individually to spacelike separated parties

*Grunhaus, Popescu, Rohrlich 1996.

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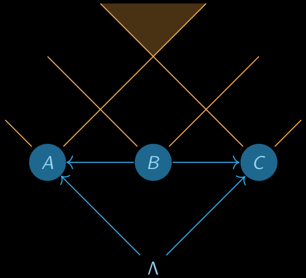

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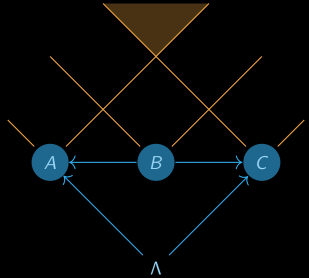

$\exists$ causal model with affects relations**:

B does not affect A or C, B affects AC

Compatibility conditions:

$$\overline{\mathcal{F}}(\mathcal{A}) \cap \overline{\mathcal{F}}(\mathcal{C}) \subseteq \overline{\mathcal{F}}(\mathcal{B})$$

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**Vilasini and Colbeck, PRA+PRL 2022. Vilasini, PhD thesis, arXiv:2102.02393.

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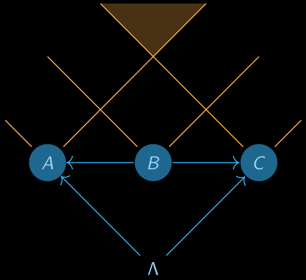

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Causal models for post-quantum "jamming" theories

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Causal loops in Minkowski space-time

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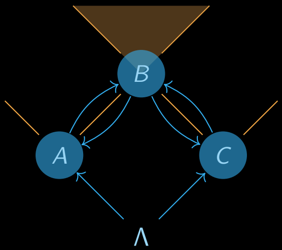

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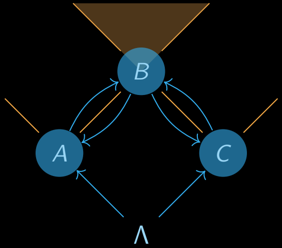

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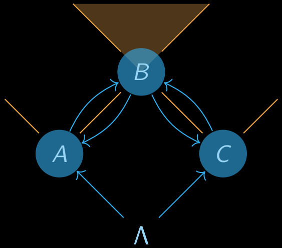
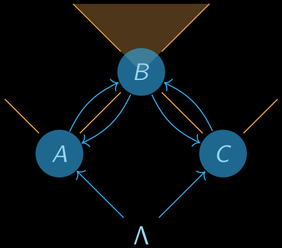
Operationally detectable causal loop embedded in Minkowski space-time without leading to superluminal signaling!



- This causal loop is only embeddable in $(1+1)$-Minkowski
- Several distinct classes of operationally detectable loops exist



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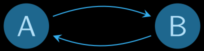
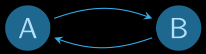
Causal loops compatibly embeddable in $(3+1)$-Minkowski space-time? Physical principles for ruling them out?

Conclusions



Physical meaning depends on space-time embedding



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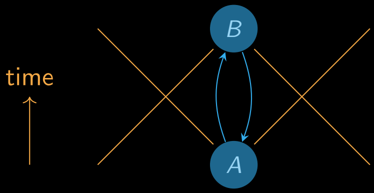
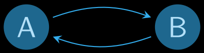
Closed timelike curve



Node $\mapsto$ space-time event
(VV and Colbeck)



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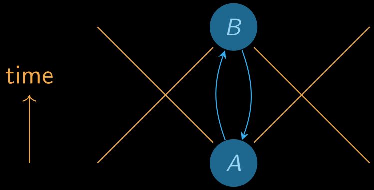

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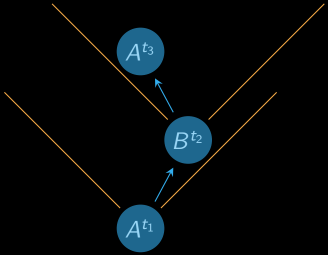
Physical feedback



Node $\mapsto$ space-time region
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Take home: Important to disentangle

- Information-theoretic vs space-time causality
- Causation, correlation, signalling
- Different principles of relativistic causality

Outlook

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  (Maarten Grothus: poster on Monday)

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THANK YOU!