QUANTUM SUPERMAPS ARE CHARACTERIZED BY LOCALITY

Matt Wilson[1,2] Giulio Chiribella[2,3,4,5], Aleks Kissinger[2]

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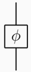
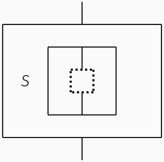
1. Department of Computer Science, University College London
2. Department of Computer Science, University of Oxford
3. HKU-Oxford Joint Laboratory for Quantum Information and Computation
4. QICI Quantum Information and Computation Initiative, Department of Computer Science, Department of Computer Science, The University of Hong Kong
5. Perimeter Institute for Theoretical Physics

Quantum Channels and Superchannels

A quantum channel:



a quantum superchannel:



G. Chiribella, P. Perinotti, and G. M. D'Ariano, Europhysics Letters 83, 30004 2008

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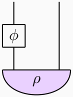

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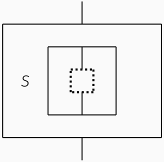

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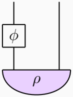

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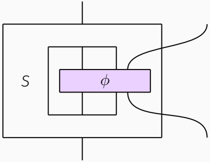
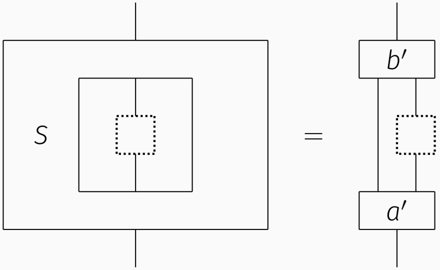


G. Chiribella, P. Perinotti, and G. M. D'Ariano, Europhysics Letters 83, 30004 2008

Circuit Decomposition

Circuit decomposition theorem:



Also true for:

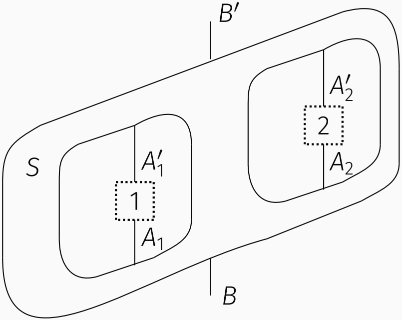
- Classical superchannels
- Pure superchannels

A. Kissinger, S. Uijlen 441 2021

W. Yokojima, M. Quintino, A. Soeda, and M. Murao, Quantum 5, 441 2021

Multi-input Supermaps

Multi-inputs:



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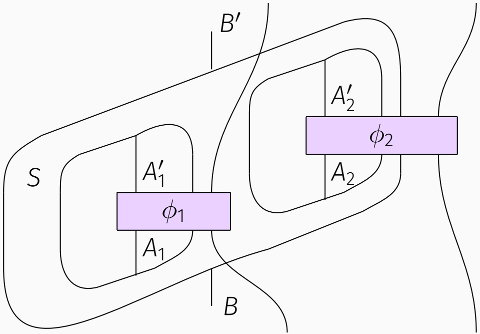

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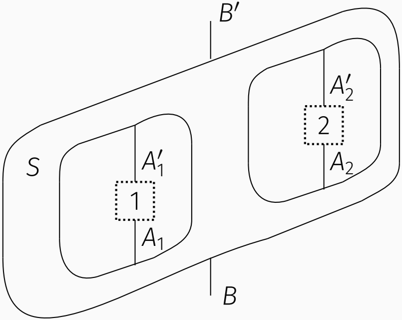
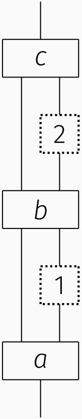
Circuit decomposition?



$\neq$



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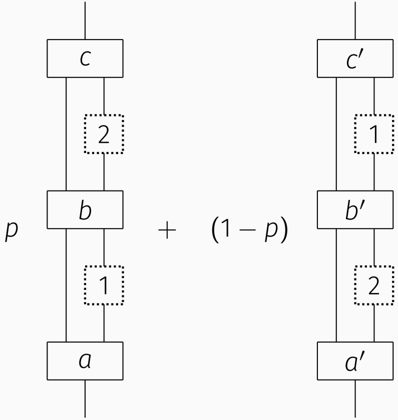

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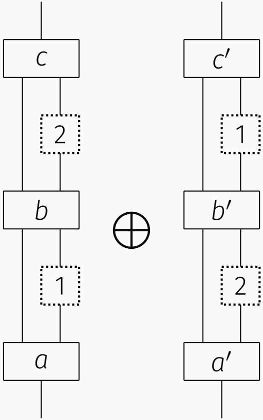

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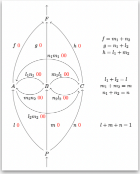
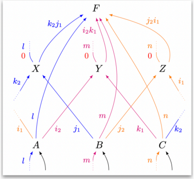
Multi-input Supermaps

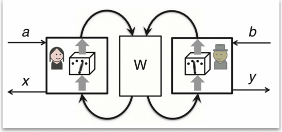
Grenoble [a]



Lugano [b]



OCB [c]

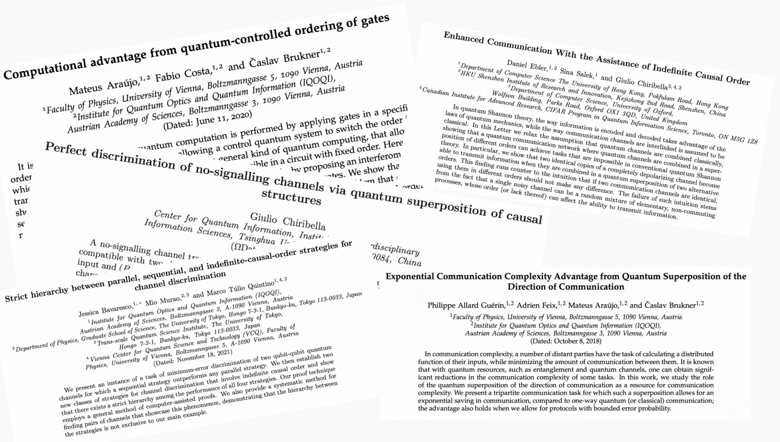


[a] J Wechs, H. Dourdent, A. Abbott, C. Branciard PRX Quantum 2, 030335 2021

[b] Ämin Baumeler, Stefan Wolf, IEEE International Symposium on Information Theory 2014

[c] O. Oreshkov, F. Costa, and Ĉ. Brukner, Nat Comms 2012

Multi-input Supermaps: Consequences



A Hole in the Framework

Current definitions use combinations of ...

G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, arXiv 2009, PRA 2013

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- Linearity

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A. Kissinger, S. Uijlen 441 2021

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A Hole in the Framework

Current definitions use combinations of ...

- Linearity
- Probability Compatibility
- Choi-Jamiolkowski Isomorphisms
- Compact closure (cups $\cup$ and caps $\cap$)

G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, arXiv 2009, PRA 2013

O. Oreshkov, F. Costa, and Ĉ. Brukner, Nat Comms 2012

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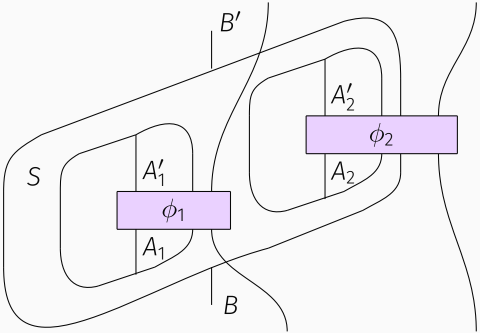
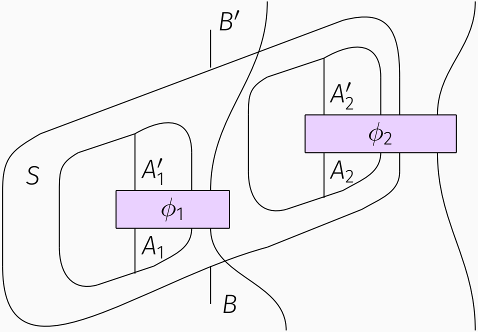
A Hole in the Framework

What kind of structure?



A Hole in the Framework

What kind of structure? Process-theoretic structure!



B. Coecke and A. Kissinger, Picturing Quantum Processes; S. MacLane, Categories for the Working Mathematician

A Hole in the Framework

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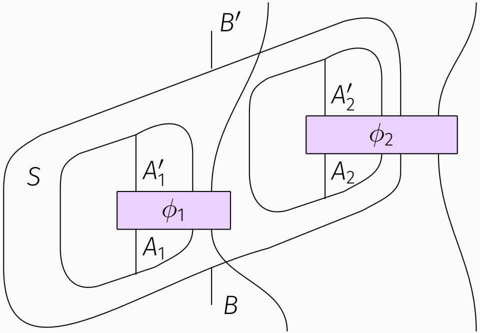
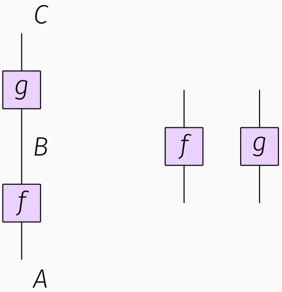


General Hilbert spaces and operational probabilistic theories?

B. Coecke and A. Kissinger, Picturing Quantum Processes; S. MacLane, Categories for the Working Mathematician

A Hole in the Framework

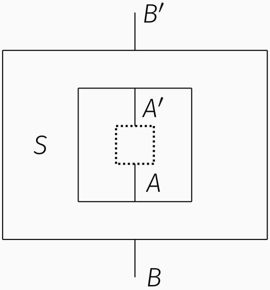
A process theory (symmetric monoidal category):



Meaning a theory **C** with sets of processes $\mathbf{C}(A, B)$:

- Sequential composition $g \circ f$ and parallel composition $f \otimes g$
- Swaps

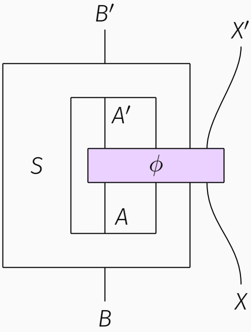
Proposed Solution



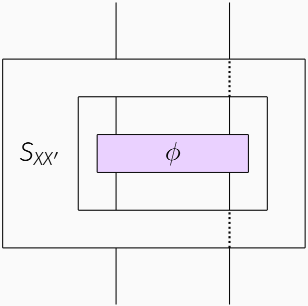
$$: \quad (A \Rightarrow A') \rightarrow (B \Rightarrow B')$$

Proposed Solution

The picture suggests a family of functions



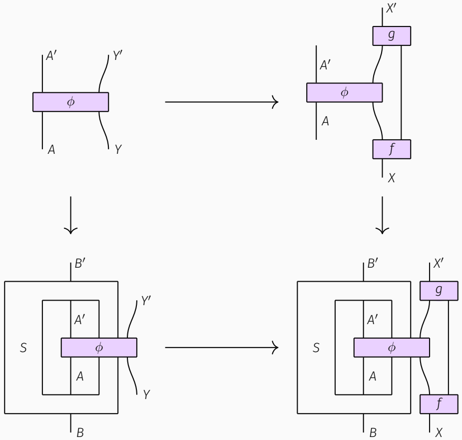
$$\approx \quad S_{X,X'}(\phi) \quad =:$$



of type

$$S_{X,X'} : \mathsf{C}(A \otimes X, A' \otimes X') \to \mathsf{C}(B \otimes X, B' \otimes X')$$

Proposed Solution



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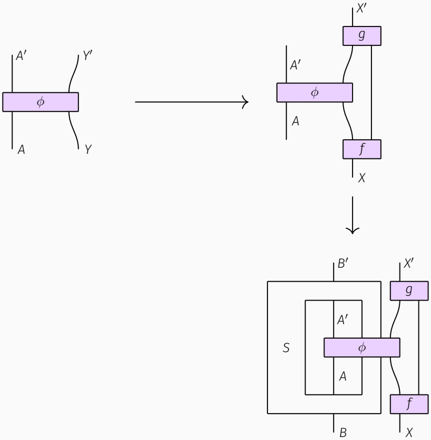

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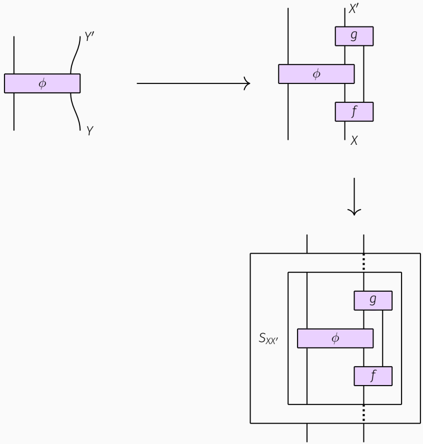

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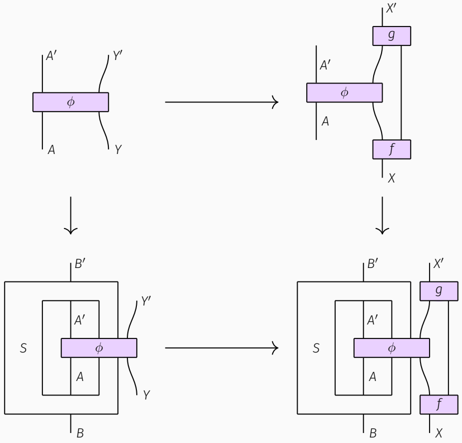

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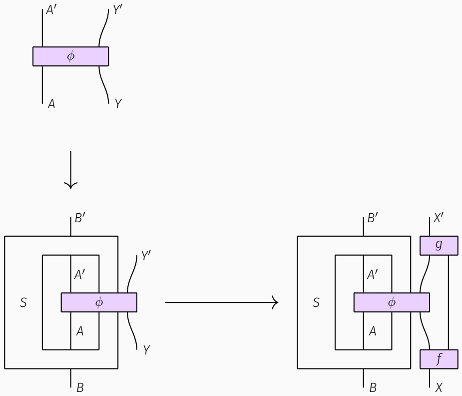

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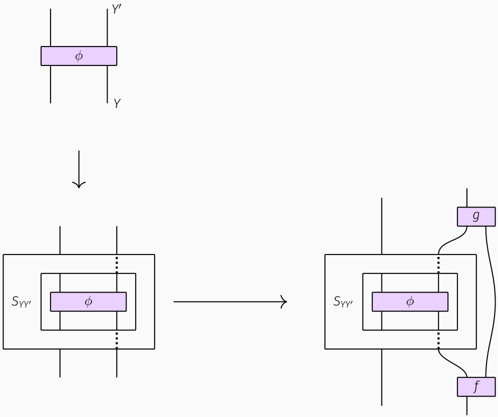
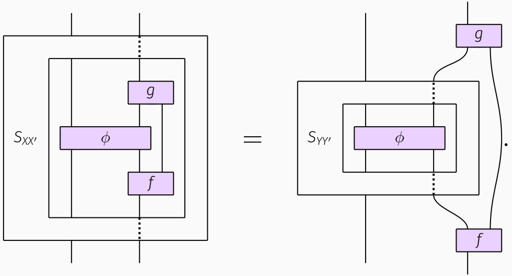


Locally Applicable Transformations

Locality: Algebraically

$$S_{X,X'}((i \otimes g) \circ (\phi \otimes i) \circ (i \otimes f)) \quad = \quad (i \otimes g) \circ (S_{Y,Y'}(\phi) \otimes i) \circ (i \otimes f)$$

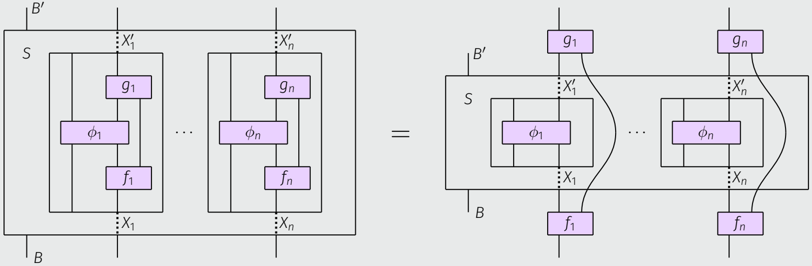
Locality: Diagrammatically



Multi-input Locally Applicable Transformations

Definition

A *lot* of type $S : (A_1 \Rightarrow A_1') \ldots (A_n \Rightarrow A_n') \longrightarrow (B \Rightarrow B')$ is a family of functions $S^{X_1' \ldots X_n'}_{X_1 \ldots X_n}$ satisfying:



Reconstruction Theorem

Theorem

There is a one-to-one correspondence between locally applicable transformations on quantum channels and deterministic quantum superchannels of type

$$(A_1 \Rightarrow A_1') \dots (A_n \Rightarrow A_n') \longrightarrow (B \Rightarrow B')$$

In categorical language, there is an equivalence of multicategories

$$\mathsf{lot}[\mathsf{QC}] \cong \mathsf{QS}$$

Categorical Supermaps

Saunders Mac Lane (Allegedly (Allegedly)):

- *I didn't invent categories to study functors; I invented them to study natural transformations.*

$$\mathsf{C}(A-, A' =) : \mathsf{C}^{op} \times \mathsf{C} \to \mathsf{SET}$$

Theorem

The quantum superchannels of type

$$(A_1 \Rightarrow A_1') \dots (A_n \Rightarrow A_n') \longrightarrow (B \Rightarrow B')$$

are the natural transformations of type

$$\times_{i=1}^{n} \mathsf{QC}(A_i -_i, A_i' =_i) \longrightarrow \mathsf{QC}(B -_1 \cdots -_n, B' =_1 \cdots =_n)$$

Outlook

We have a bare-minimum axiom for supermaps on any theory of processes:

- Arbitrary Hilbert spaces?
- Post-quantum causal structures?
- Reconstruction for all of Higher-Order Quantum Theory?

this axiom can be explained to:

- Category theorists (natural transformations)
- Your friends (circuit diagrams)

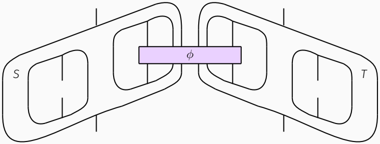
the only concept needed is the purely compositional one of **local applicability**.

Thank-You for Listening!

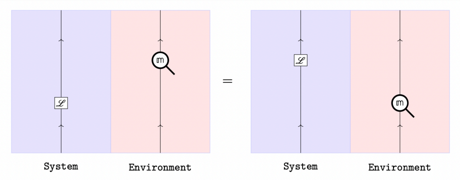
https://arxiv.org/abs/2205.09844

Follow-on Projects

- Free Polycategories for Unitary Supermaps of Arbitrary Dimension

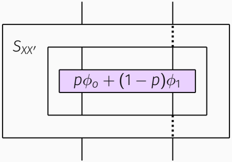


- On the Origin of Linearity and Unitarity in Quantum Theory



M. Wilson, G. Chiribella, arXiv 2022; M. Wilson, N. Ormrod, arXiv 2023

Convex Linearity



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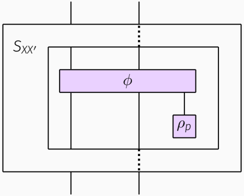

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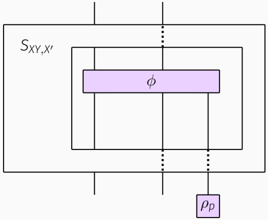
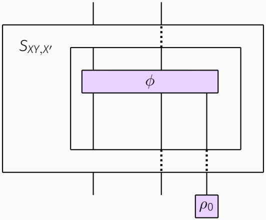
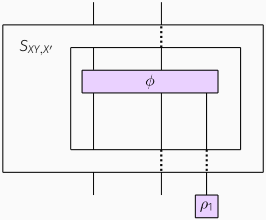


$= \quad p_0$



$+ \quad p_1$



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Convex Linearity

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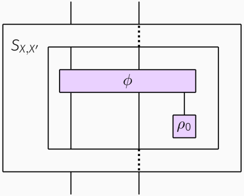

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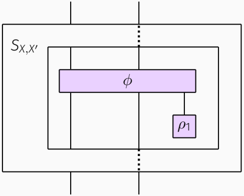
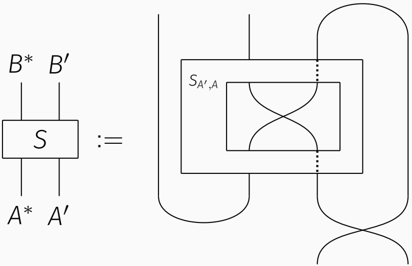


Using cups and caps



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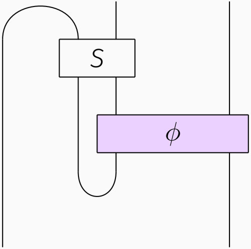

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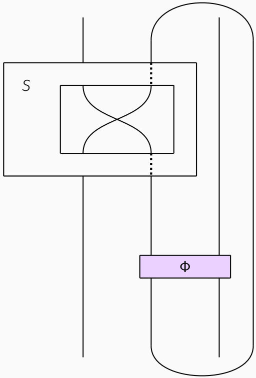

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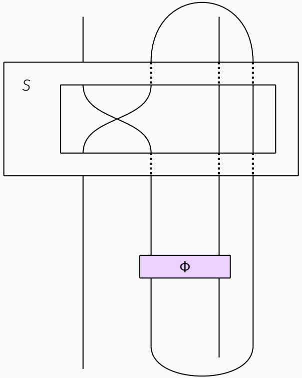

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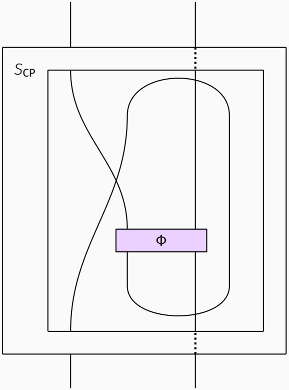

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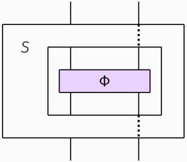
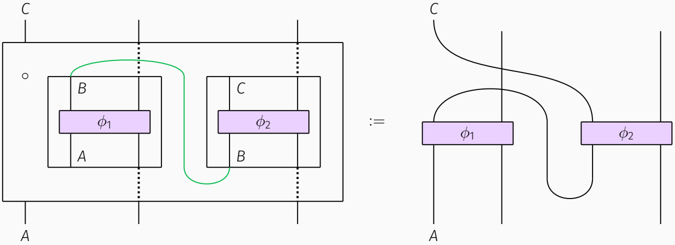
$=$ 

Enriched Structure for Supermaps

Locally-applicable transformations enrich the category on which they act



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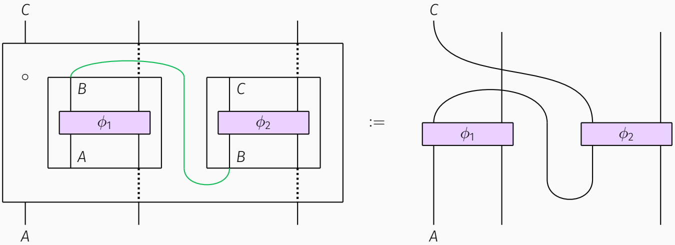

Formally have constructed a $\mathrm{lot}[\mathbf{C}]$-category $\mathbf{C}$

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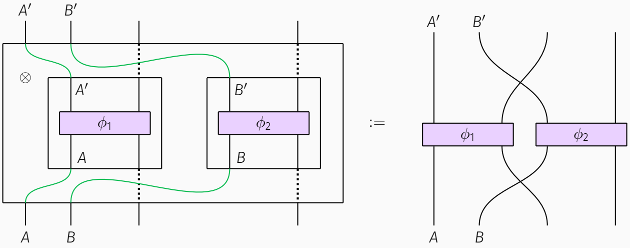

Formally have constructed a **lot[C]**-monoidal category **C**