

QUANTUM SUPERMAPS ARE CHARACTERIZED BY LOCALITY

Matt Wilson^{1,2} Giulio Chiribella^{2,3,4,5}, Aleks Kissinger²

July 21, 2023

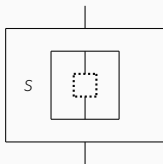
1. Department of Computer Science, University College London
2. Department of Computer Science, University of Oxford
3. HKU-Oxford Joint Laboratory for Quantum Information and Computation
4. QICI Quantum Information and Computation Initiative, Department of Computer Science, Department of Computer Science, The University of Hong Kong
5. Perimeter Institute for Theoretical Physics

QUANTUM CHANNELS AND SUPERCHANNELS

A quantum channel:



a quantum superchannel:



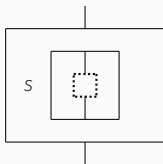
G. Chiribella, P. Perinotti, and G. M. D'Ariano, Europhysics Letters 83, 30004 2008

QUANTUM CHANNELS AND SUPERCHANNELS

A quantum channel:



a quantum superchannel:



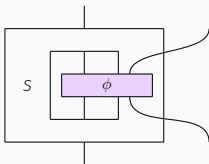
G. Chiribella, P. Perinotti, and G. M. D'Ariano, Europhysics Letters 83, 30004 2008

QUANTUM CHANNELS AND SUPERCHANNELS

A quantum channel:



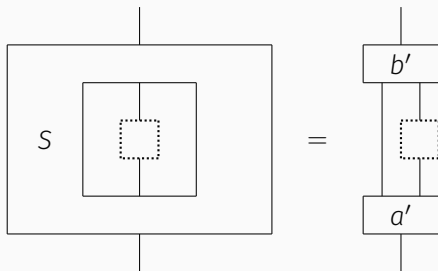
a quantum superchannel:



G. Chiribella, P. Perinotti, and G. M. D'Ariano, Europhysics Letters 83, 30004 2008

CIRCUIT DECOMPOSITION

Circuit decomposition theorem:

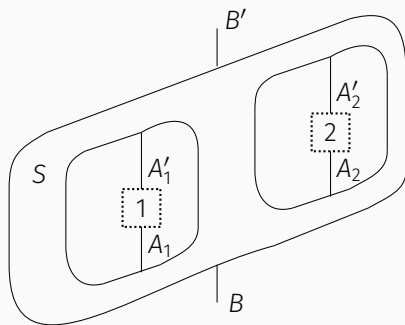


Also true for:

- Classical superchannels
- Pure superchannels

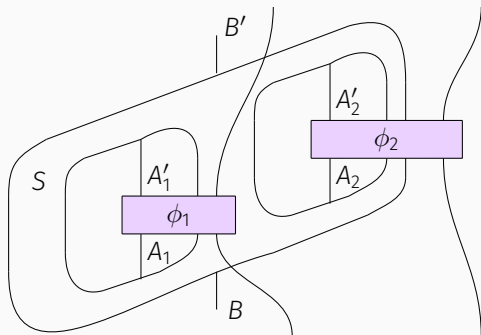
MULTI-INPUT SUPERMAPS

Multi-inputs:



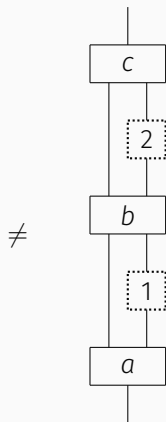
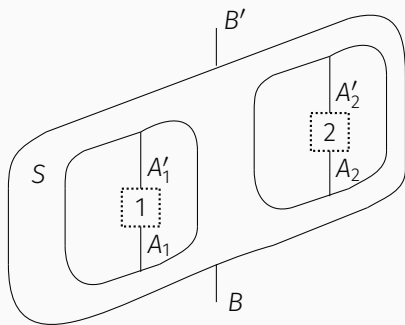
MULTI-INPUT SUPERMAPS

Multi-inputs:



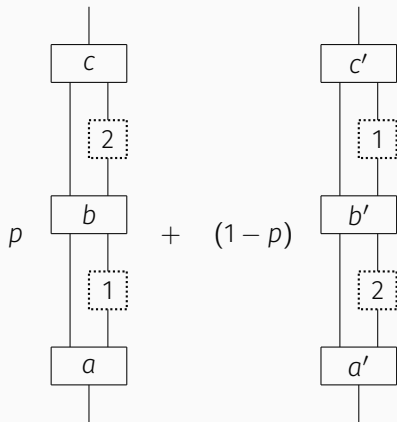
MULTI-INPUT SUPERMAPS

Circuit decomposition?



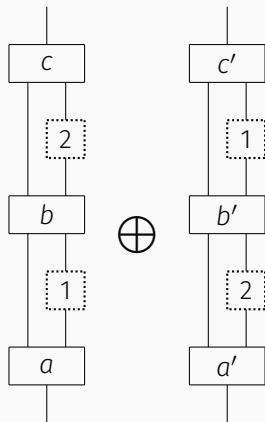
MULTI-INPUT SUPERMAPS

Circuit decomposition?



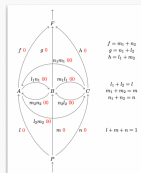
MULTI-INPUT SUPERMAPS

Circuit decomposition?

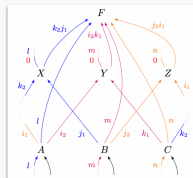


MULTI-INPUT SUPERMAPS

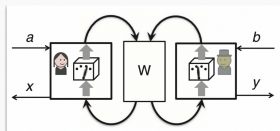
Grenoble [a]



Lugano [b]



OCB [c]



[a] J Wechs, H. Dourdent, A. Abbott, C. Branciard PRX Quantum 2, 030335 2021

[b] Āmin Baumeler, Stefan Wolf, IEEE International Symposium on Information Theory 2014

[c] O. Oreshkov, F. Costa, and Ā. Brukner, Nat Comms 2012

MULTI-INPUT SUPERMAPS: CONSEQUENCES

Computational advantage from quantum-controlled ordering of gates

Matheus Araújo,^{1,2} Fabio Costa,^{1,2} and Časlav Brukner^{1,2}
¹Faculty of Physics, University of Vienna, Boltzmannngasse 5, 1090 Vienna, Austria
²Institute for Quantum Optics and Quantum Information (IQOQI),
Austrian Academy of Sciences, Boltzmannngasse 3, 1090 Vienna, Austria
(Dated: June 11, 2020)

Quantum computation is performed by applying gates in a specific order, allowing a control quantum system to switch the order of general kind of quantum computing, that allow one to propose an interferometric structure in a circuit with fixed order. Here we show that

Perfect discrimination of no-signalling channels via quantum superposition of causal structures

Giulio Chiribella
Center for Quantum Information, Institute for
Information Sciences, Tsinghua University
(Beijing, P.R. China)

A no-signalling channel compatible with two input and P channel discrimination

Strict hierarchy between parallel, sequential, and indefinite-causal-order strategies for channel discrimination

Jesús Barreiro,^{1,2} Milo Murao,^{2,3} and Marco Túlio Quintino^{2,4,5}
¹Institute for Quantum Optics and Quantum Information (IQOQI),
Austrian Academy of Sciences, Boltzmannngasse 3, A-1090 Vienna, Austria
²Department of Physics, University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan
³Trans-scale Quantum Science Institute, The University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan
⁴Vienna Center for Quantum Science and Technology (VCQ), Faculty of Physics, University of Vienna, Boltzmannngasse 5, A-1090 Vienna, Austria
(Dated: November 18, 2021)

We present an instance of a task of minimum-error discrimination of two qubit-qubit quantum channels for which a sequential strategy outperforms any parallel strategy. We then establish two new classes of strategies for channel discrimination that involve indefinite causal order and show that these exhibit a strict hierarchy among the performance of all four strategies. Our proof technique employs a general method of convex-optimization. We also provide a systematic method for finding pairs of channels that showcase this phenomenon, demonstrating that the hierarchy between the strategies is not exclusive to our specific example.

Enhanced Communication With the Assistance of Indefinite Causal Order

Daniel Ebler,^{1,2} Sina Salek,¹ and Giulio Chiribella^{2,3,4,5}
¹Department of Computer Science, The University of Hong Kong, Pokfulam Road, Hong Kong
²HKU Shenzhen Institute of Research and Innovation, Kowloon 3rd Road, Shenzhen, China
³Wolfson Building, Faculty of Computer Science, University of Oxford, Wolfson Building, Parks Road, Oxford OX1 2QJ, United Kingdom
⁴Canadian Institute for Advanced Research, CIFAR Program in Quantum Information Science, Toronto, ON M5G 1Z5
⁵Department of Computer Science, University of Toronto, 279 Spadina Avenue, Toronto, ON M5S 2E4, Canada

In quantum Shannon theory, the way information is encoded and decoded takes advantage of the classical nature of quantum resources, while the way communication channels are identified is assumed to be classical. In this Letter we relax this assumption that quantum channels are identified classically, showing that a quantum communication network where quantum channels are combined is able to transmit information when they are combined in a superposition of different causal orders. In particular, we show that two identical copies of a completely depolarizing channel become useful for transmitting information when they are combined in a superposition of two alternative causal orders. This finding runs counter to the intuition that if two communication channels are identical, then the fact that a single copy should not make any difference. The failure of such intuition stems from the fact that a single copy channel can be a random mixture of elementary, non-communicating processes, whose order (or lack thereof) can affect the ability to transmit information.

disciplinarity
7084, China

Exponential Communication Complexity Advantage from Quantum Superposition of the Direction of Communication

Philippe Allard Guérin,^{1,2} Adrien Feix,^{1,2} Matheus Araújo,^{1,2} and Časlav Brukner^{1,2}
¹Faculty of Physics, University of Vienna, Boltzmannngasse 5, 1090 Vienna, Austria
²Institute for Quantum Optics and Quantum Information (IQOQI),
Austrian Academy of Sciences, Boltzmannngasse 3, 1090 Vienna, Austria
(Dated: October 8, 2018)

In communication complexity, a number of distant parties have the task of calculating a distributed function of their inputs, while minimizing the amount of communication between them. It is known that with quantum resources, such as entanglement and quantum channels, one can obtain significant reductions in the communication complexity of some tasks. In this work, we study the role of the quantum superpositions of the direction of communication as a resource for communication complexity. We present a tripartite communication task for which such a superposition allows for an exponential saving in communication, compared to one-way quantum (or classical) communication; the advantage also holds when we allow for protocols with bounded error probability.

Current definitions use combinations of ...

G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, arXiv 2009, PRA 2013

O. Oreshkov, F. Costa, and Č. Brukner, Nat Comms 2012

A. Kissinger, S. Uijlen 441 2021

Current definitions use combinations of ...

- Linearity

G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, arXiv 2009, PRA 2013

O. Oreshkov, F. Costa, and Č. Brukner, Nat Comms 2012

A. Kissinger, S. Uijlen 441 2021

Current definitions use combinations of ...

- Linearity
- Probability Compatibility

G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, arXiv 2009, PRA 2013

O. Oreshkov, F. Costa, and Č. Brukner, Nat Comms 2012

A. Kissinger, S. Uijlen 441 2021

Current definitions use combinations of ...

- Linearity
- Probability Compatibility
- Choi-Jamiolkowski Isomorphisms

G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, arXiv 2009, PRA 2013

O. Oreshkov, F. Costa, and Č. Brukner, Nat Comms 2012

A. Kissinger, S. Uijlen 441 2021

Current definitions use combinations of ...

- Linearity
- Probability Compatibility
- Choi-Jamiolkowski Isomorphisms
- Compact closure (cups \cup and caps \cap)

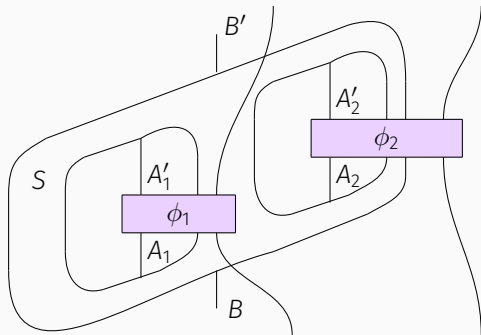
G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, arXiv 2009, PRA 2013

O. Oreshkov, F. Costa, and Č. Brukner, Nat Comms 2012

A. Kissinger, S. Uijlen 441 2021

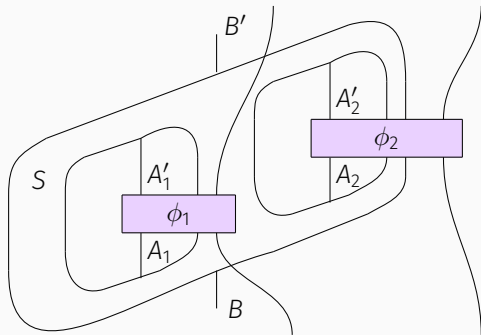
A HOLE IN THE FRAMEWORK

What kind of structure?



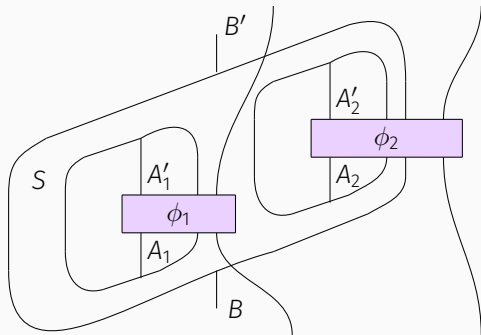
A HOLE IN THE FRAMEWORK

What kind of structure? Process-theoretic structure!



A HOLE IN THE FRAMEWORK

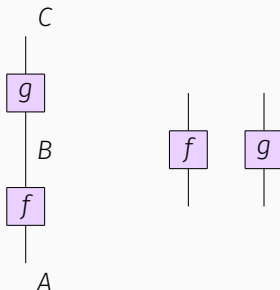
What kind of structure? Process theoretic structure!



General Hilbert spaces and operational probabilistic theories?

A HOLE IN THE FRAMEWORK

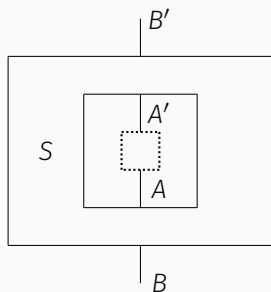
A process theory (symmetric monoidal category):



Meaning a theory \mathbf{C} with sets of processes $\mathbf{C}(A, B)$:

- Sequential composition $g \circ f$ and parallel composition $f \otimes g$
- Swaps

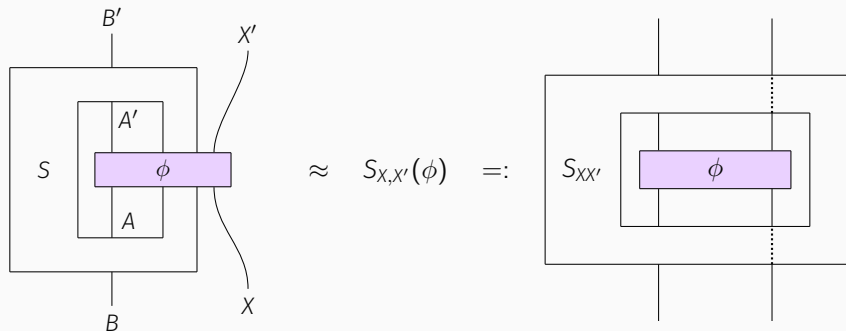
PROPOSED SOLUTION



$$: (A \Rightarrow A') \rightarrow (B \Rightarrow B')$$

PROPOSED SOLUTION

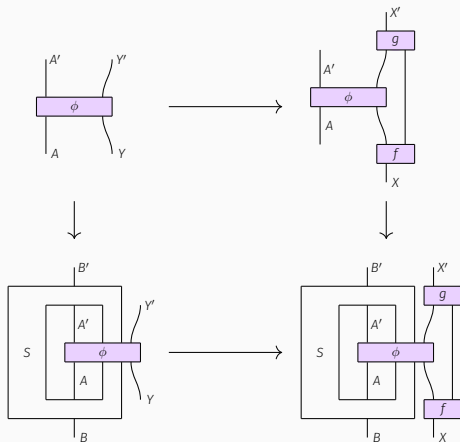
The picture suggests a family of functions



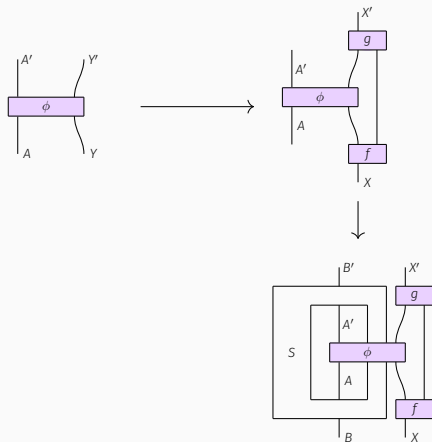
of type

$$S_{X,X'} : \mathbf{C}(A \otimes X, A' \otimes X') \rightarrow \mathbf{C}(B \otimes X, B' \otimes X')$$

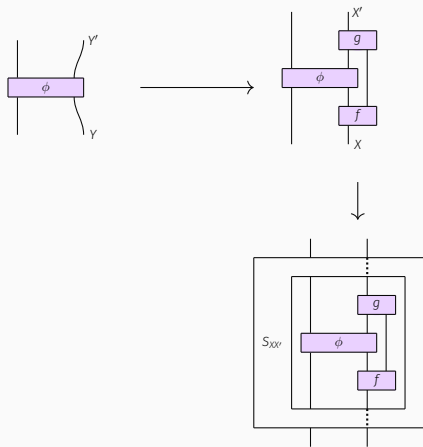
PROPOSED SOLUTION



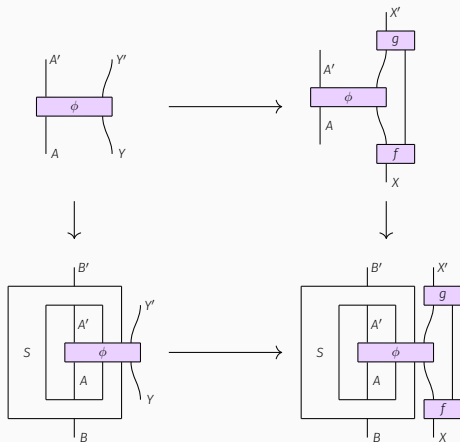
PROPOSED SOLUTION



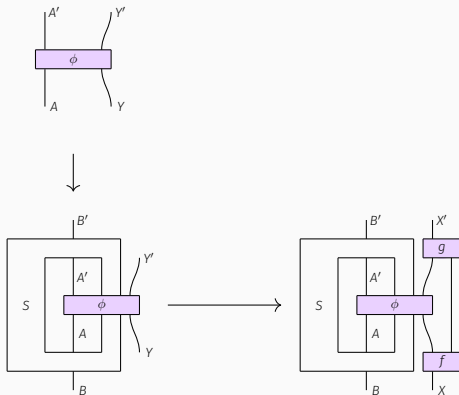
PROPOSED SOLUTION



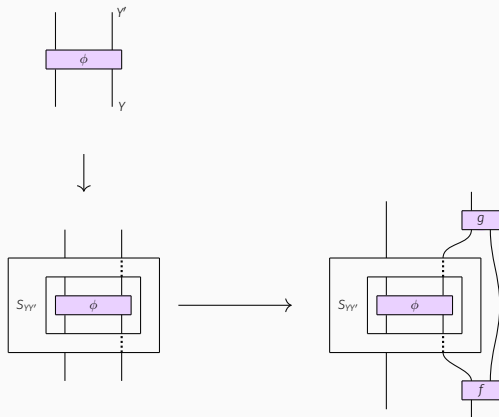
PROPOSED SOLUTION



PROPOSED SOLUTION



PROPOSED SOLUTION

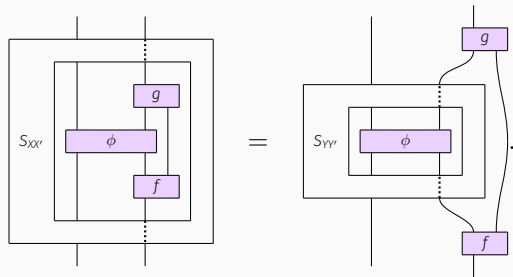


LOCALLY APPLICABLE TRANSFORMATIONS

Locality: Algebraically

$$S_{X,X'}((i \otimes g) \circ (\phi \otimes i) \circ (i \otimes f)) = (i \otimes g) \circ (S_{Y,Y'}(\phi) \otimes i) \circ (i \otimes f)$$

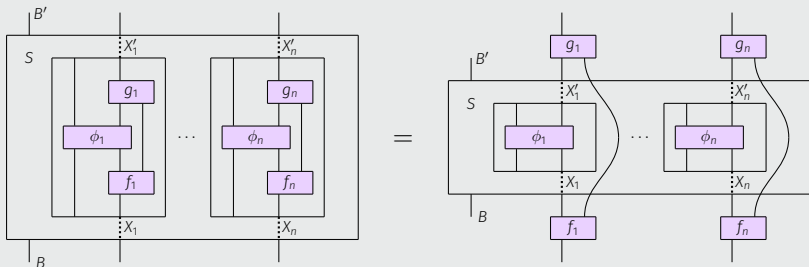
Locality: Diagrammatically



MULTI-INPUT LOCALLY APPLICABLE TRANSFORMATIONS

Definition

A lot of type $S : (A_1 \Rightarrow A'_1) \dots (A_n \Rightarrow A'_n) \longrightarrow (B \Rightarrow B')$ is a family of functions $S_{X_1 \dots X_n}^{X'_1 \dots X'_n}$ satisfying:



RECONSTRUCTION THEOREM

Theorem

There is a one-to-one correspondence between locally applicable transformations on quantum channels and deterministic quantum superchannels of type

$$(A_1 \Rightarrow A'_1) \dots (A_n \Rightarrow A'_n) \longrightarrow (B \Rightarrow B')$$

In categorical language, there is an equivalence of multicategories

$$\text{lot}[\text{QC}] \cong \text{QS}$$

Saunders Mac Lane (Allegedly (Allegedly)):

- *I didn't invent categories to study functors; I invented them to study natural transformations.*

$$\mathbf{C}(A \dashv, A' \dashv) : \mathbf{C}^{op} \times \mathbf{C} \rightarrow \mathbf{SET}$$

Theorem

The quantum superchannels of type

$$(A_1 \Rightarrow A'_1) \dots (A_n \Rightarrow A'_n) \longrightarrow (B \Rightarrow B')$$

are the natural transformations of type

$$\times_{i=1}^n \mathbf{QC}(A_i \dashv_i, A'_i \dashv_i) \longrightarrow \mathbf{QC}(B \dashv_{-1} \dots \dashv_{-n}, B' \dashv_{=1} \dots \dashv_{=n})$$

We have a bare-minimum axiom for supermaps on any theory of processes:

- Arbitrary Hilbert spaces?
- Post-quantum causal structures?
- Reconstruction for all of Higher-Order Quantum Theory?

this axiom can be explained to:

- Category theorists (natural transformations)
- Your friends (circuit diagrams)

the only concept needed is the purely compositional one of **local applicability**.

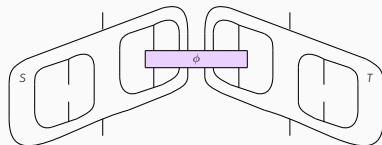
THANK-YOU FOR LISTENING!

<https://arxiv.org/abs/2205.09844>

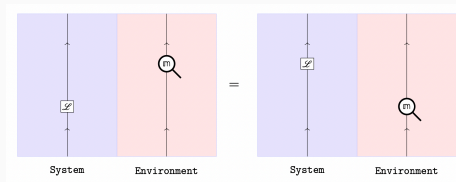


FOLLOW-ON PROJECTS

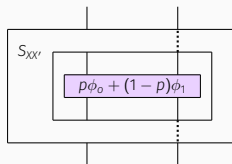
- Free Polycategories for Unitary Supermaps of Arbitrary Dimension



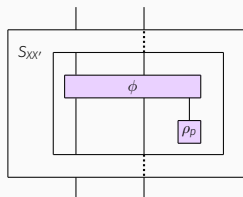
- On the Origin of Linearity and Unitarity in Quantum Theory



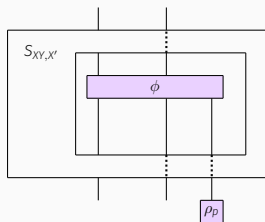
CONVEX LINEARITY



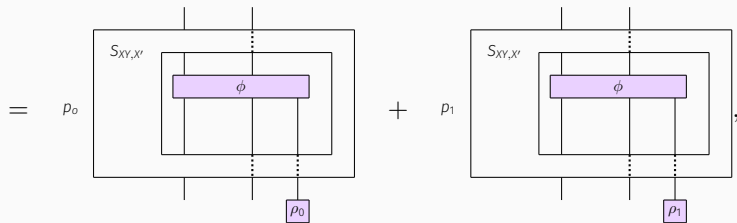
CONVEX LINEARITY



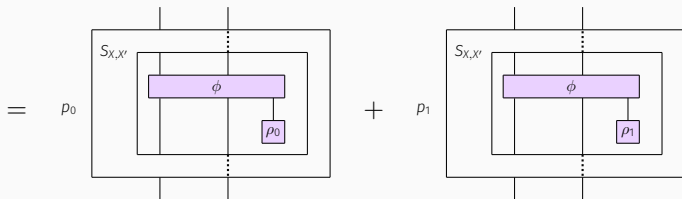
CONVEX LINEARITY



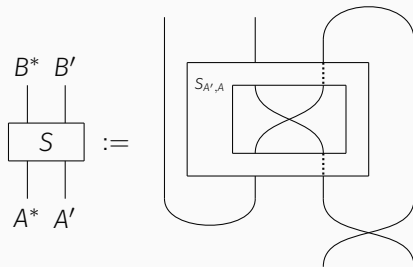
CONVEX LINEARITY



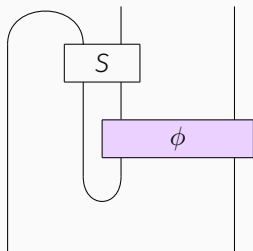
CONVEX LINEARITY



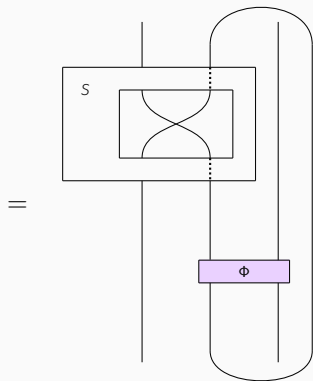
USING CUPS AND CAPS



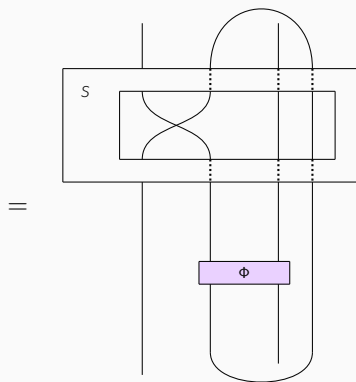
USING CUPS AND CAPS



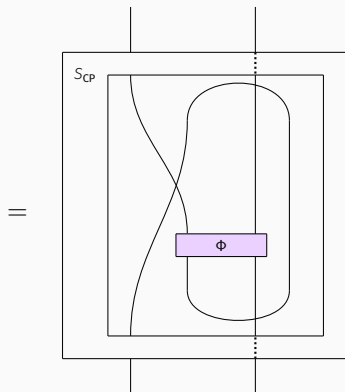
USING CUPS AND CAPS



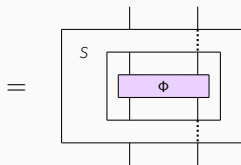
USING CUPS AND CAPS



USING CUPS AND CAPS

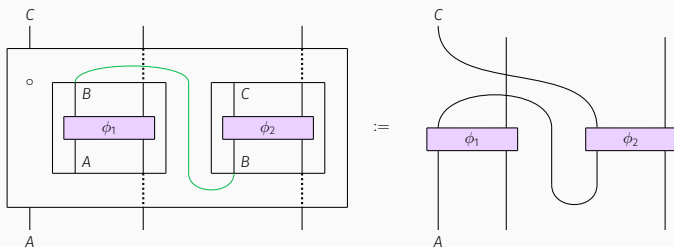


USING CUPS AND CAPS



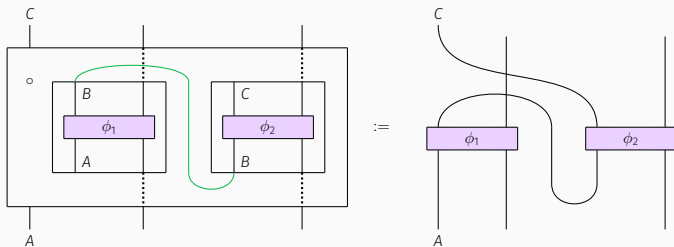
ENRICHED STRUCTURE FOR SUPERMAPS

Locally-applicable transformations enrich the category on which they act



ENRICHED STRUCTURE FOR SUPERMAPS

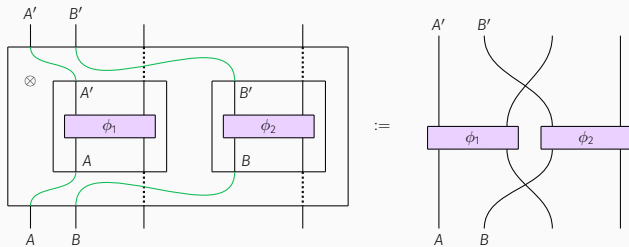
Locally-applicable transformations enrich the category on which they act



Formally have constructed a $\mathbf{lot}[\mathbf{C}]$ -category \mathbf{C}

ENRICHED STRUCTURE FOR SUPERMAPS

Locally-applicable transformations enrich the category on which they act



Formally have constructed a $\mathbf{lot}[\mathbf{C}]$ -monoidal category \mathbf{C}